DOCUMENT RESUME

ED 053 966 24 SE 012 222

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TITLE The Development of an Individualized Instructional Program in Beginning College Mathematics Utilizing Computer Based Resource Units. Final Report.
INSTITUTION State Univ. of New York, Brockport. Coll. at Brockport.
SPONS AGENCY Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO BR-0-B-110
PUB DATE Jun 71
GRANT OEG-2-700045(509)
NOTE 79p.

EDRS PRICE EDRS Price MF-$0.65 HC-$3.29
DESCRIPTORS Calculus, *College Mathematics, *Computer Assisted Instruction, Individual Instruction, *Individualized Instruction, Individualized Programs, *Mathematics Instruction, *Programed Materials, Set Theory, Trigonometry

ABSTRACT

Reported is an attempt to develop and evaluate an individualized instructional program in pre-calculus college mathematics. Four computer based resource units were developed in the areas of set theory, relations and function, algebra, trigonometry, and analytic geometry. Objectives were determined by experienced calculus teachers, and multiple-choice questions were written for each objective. Programmed instructional materials were selected for use by the students. Computer programs were written for each unit which diagnosed student difficulties and provided printed outputs of instructional materials for each objective not satisfied by the student. One of two college pre-calculus classes used the resource units while the other class acted as a control group. No significant differences in achievement were found. A general computer program was written and is reported which allows an instructor to input objectives, criteria for satisfying objectives and instructional materials for each objective. Appendixed are the objectives, pretests, sample output, and program listings for each of the four units. (Author/JG)

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FINAL REPORT
Project No. 0B110
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THE DEVELOPMENT OF AN INDIVIDUALIZED INSTRUCTIONAL PROGRAM IN BEGINNING COLLEGE MATHEMATICS UTILIZING COMPUTER BASED RESOURCE UNITS

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The research reported herein was performed pursuant to a grant (grant or contract) with the Office of Education, U.S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy.

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CONTENTS

Summary.....1

Introduction.....2

Methods.....6

Results.....8

Conclusions and Recommendations.....14

Appendix.....16

Tables

I. Control Group Correlation Matrix.....10

II. Experimental Group Correlation Matrix....10

III. Means and Standard Deviations for Achievement Variables.....11

IV. Data on Resource Unit Pretests.....11

V. Analysis of Variance.....12

VI. Histograms for Calculus Pretest.....13

SUMMARY

The purpose of this project was to develop and evaluate an individualized instructional program in pre-calculus mathematics. Computer based resource units were developed which produce individualized instructional units based upon the student's background and understanding of each pre-calculus topic.

Experienced calculus teachers determined objectives which should be satisfied by students entering calculus in each of four pre-calculus units: Sets, Relations and Functions; Algebra; Trigonometry; and Analytic Geometry. Multiple-choice questions were then written for each objective and instructional materials, primarily programmed materials, were selected which would enable a student to satisfy each objective. For each pre-calculus unit a computer program was written which takes student responses to the multiple-choice questions as input, determines which objectives the student has not satisfied, and provides printed output of instructional materials for each objective not satisfied.

The resource units were used with students enrolled in one of two pre-calculus classes. In general, achievement of the class using the resource units did not differ significantly from the achievement of the control class. The unit pretests revealed extreme variability in student understanding of pre-calculus mathematics, strongly supporting an individualized instructional approach.

To make the instructional system adaptable to other courses, a general computer program was written which allows an instructor to input objectives, criteria for satisfying objectives and instructional materials for each objective. This general program is compatible with test scoring machines which provide punched output and could be used effectively by an instructor who encounters considerable variability in student preparation for a given course.

Introduction

Students enter calculus courses with considerable variability in mathematical maturity. This is due in part to the time spent in previous study of mathematics and to the nature of mathematics courses previously studied. Because of the variability in high school mathematics courses beyond the three year college preparatory program, many students with four years of high school mathematics are inadequately prepared in some topics of pre-calculus mathematics. Such variability of preparation is clearly revealed when the calculus student encounters the definition of function. Successful study of differential and integral calculus depends heavily upon the student's ability to work with functions and upon his competency in algebra and analytic geometry. The problem, therefore, is how to assure that students begin the study of calculus with an adequate understanding of the essential pre-calculus mathematics topics without requiring all students to take the same pre-calculus course. It is unreasonable to require a student to take an entire course when he may be deficient in only one topic, however it may be unwise for the student to enter calculus with such a deficiency.

The purpose of this project was to develop and evaluate an individualized instructional program in pre-calculus mathematics. The project focused upon four units of pre-calculus mathematics: Sets, Relations and Functions; Analytic Geometry; Algebra; Elementary Functions.

Specific objectives of the project were:

(1) To develop instruments for measuring student understanding of each pre-calculus topic,

(2) To determine the effectiveness of the instruments in identifying weaknesses which lead to failure in calculus,

(3) To design individual instructional units based upon the student's understanding of each pre-calculus topic,

(4) To evaluate the use of the instructional units with students in a pre-calculus course,

(5) To prepare a manual of instruction for using and modifying the pre-calculus units.

Recognition of the fact that a person learns as an individual rather than as a "typical" member of a group is the first step toward individualized instruction. Ideally an instructor would: (1) determine each student's understanding of a topic, (2) prescribe an appropriate program through which the student would meet the objectives of the topic and (3) manage and evaluate the student's progress through the topics. Such an individualized approach would involve the instructor in exhausting and needless repetition. The need for extensive memory and adaptability to repetitive processes suggest that the computer could be used in this individualized approach. This is precisely the purpose of a computer based resource unit.

A resource unit is here defined as a collection of suggested learning activities and materials, organized around specific objectives on a given topic, which the instructor may use to help him individualize a teaching unit. In a computer based resource unit the computer serves as a storehouse of information on resource materials and selects from this storehouse an individualized instructional unit for each student.

Extensive research on Computer Assisted Instruction (CAI), which involves direct student-computer interaction, has been done at a number of universities throughout the United States. Comparable research on the use of the computer for the management of instruction and specifically, computer based resource units, does not exist.[1] Perhaps the most extensive work on the development of computer based resource units has been done at the Curriculum Center of the Erie County Board of Cooperative Educational Services in conjunction with The Center of Curriculum Planning, SUNY at Buffalo, New York.[2] To date nineteen units have been developed and are currently being used by elementary and secondary schools in Wester New York. These units were prepared by experienced teachers working with curriculum specialists and are subject to revision through the suggestions made by the current users. The continued use which these resource units are receiving is an indication of the success of computer based resource units in elementary and secondary schools.

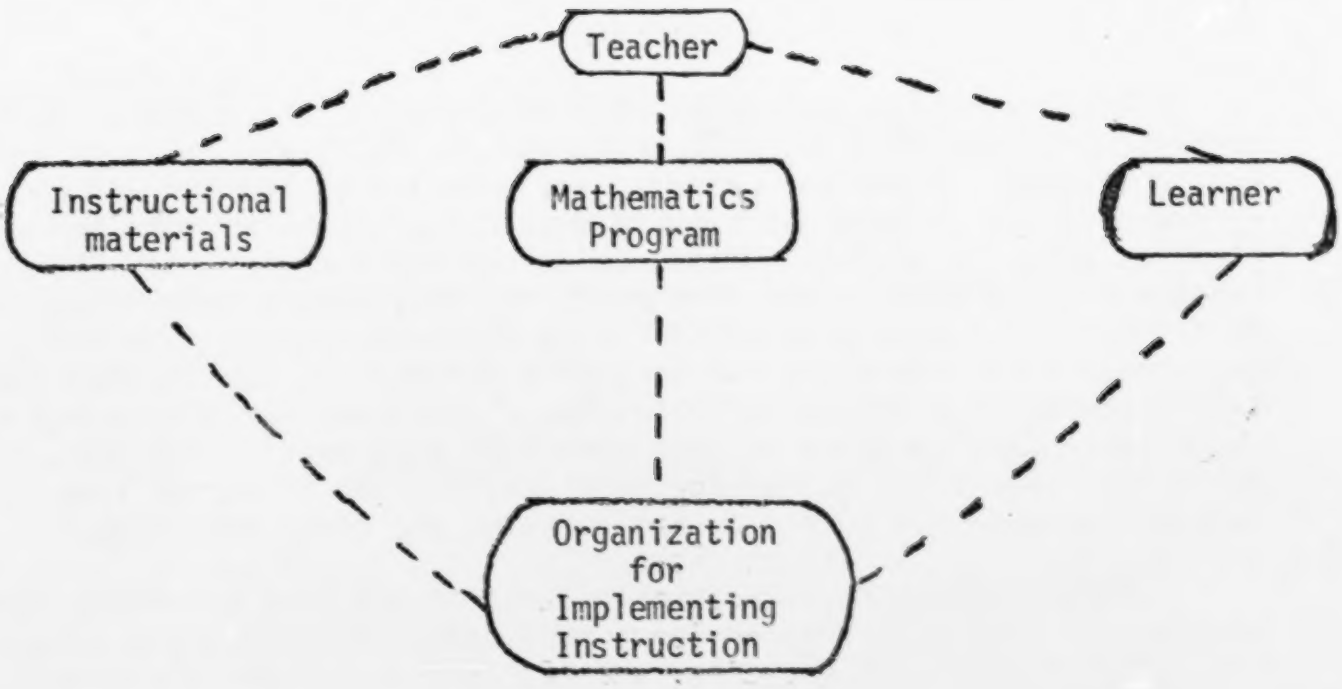
[1]Eisele, James E., "Computers in Curriculum Planning," Educational Technology, Vol. VII, No. 22, (Nov. 30, 1967), p 9-16.

[2]Eisele, James E. and Robert S. Harnack, "Improving Teacher Decision-Making and Individualization of Instruction," The Quarterly, Vol. XVII, No. 4 (May 1969), p.8.

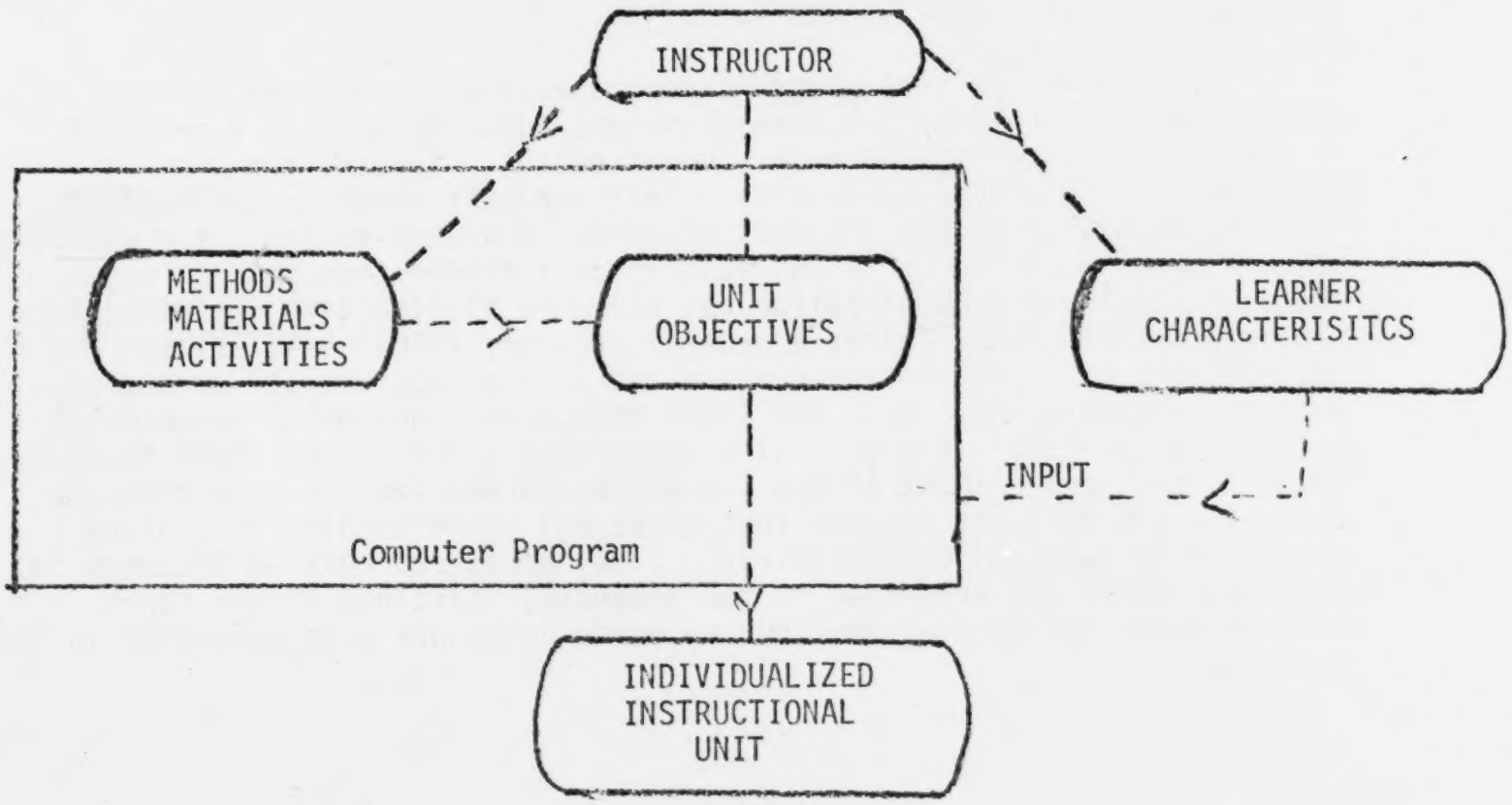
From the extensive research on individualized instruction one concludes that mathematics is a discipline which should lend itself to computerized individualization of instruction, and that such individualized instruction should be a most effective alternative to group instruction when considerable variability in background exists. It thus seems appropriate to attack the problem of bringing students with extreme variability in background to the same level of achievement in pre-calculus mathematics through the use of computer based resource units.

The activities of the project focused on the application of a recently suggested research model to the problem of developing individualized instructional units. In a recent article on learning theory and research in mathematics education,[1] J. Fred Weaver suggests five necessary components to be considered in establishing any comprehensive framework for research in mathematics education. These are identified as: (A) The mathematics program, (B) The learner, (C) The teacher, (D) Instructional materials, methods, activities, and (E) Organization for implementing instruction.

[1]Weaver, J. Fred, "Using Theories of Learning and Instruction in Elementary School Mathematics Research," The Arithmetic Teacher, Vol. XVI, No. 5 (May, 1969), p. 379-382.



This framework provided a model for the development of computer based resource units.



Methods

Resource Units Developed - During the first component of this research project, June 15, 1970 to August 31, 1970, the resource units were developed. Three consultants, all experienced teachers of calculus, worked with the project director in developing resource units in Algebra, Analytic Geometry, Sets, Relations and Functions and Elementary Functions. For each of the four units the consultants determined objectives which should be satisfied by students preparing to study calculus. With objectives for each unit determined, the project staff then prepared five option multiple-choice questions for evaluating each objective. Usually three or four questions were written for each objective. The fifth option for each question was "I do not know." This option was used to discourage guessing and blank responses.

After evaluating a large assortment of teaching materials for each precalculus unit, the project staff selected appropriate resource materail for each unit objective. First, resource materials were selected on the basis of their independent treatment of the objective in question and, second, on the basis of readability and organization. Copies of objectives and multiple choice questions for each precalculus unit are appended to this report.

Computer Resource Programs Written - During the summer component of the project, but after the resource units were developed, the project director and a computer programmer wrote computer programs for each of the pre-calculus resource units. These programs were designed to take a student's answers on a unit pretest as input, determine which objectives the student has not satisfied, and provide printed output of instructional material for each objective not satisfied.

After the four unit programs were operational, several modifications were made. First, a subroutine was added to provide a printout of a unit bibliography with each student output. Second, the unit programs were made compatable with a test analysis program, SUPERGRADER. This compatability allows the unit programs to accept as input the multiple punched cards produced by an IBM 1230 Optical Reader from student answer sheets. This second modification was far more difficult to implement than had been anticipated. Third, a general program, INFOS, was written which includes the features mentioned above and allows input of all parameters, headings and messages. Data for INFOS thus determines which resource unit is being processed. This general program uses a subroutine INFO2 to process student responses. INFOS allows a question to be used for more than one objective and does not require that questions corresponding to a given objective be numbered consecutively. Flow charts and FORTRAN listings for INFOS and INFO2 are presented in the appendix. Listings of the input control cards for each of the four resource units are also presented in the appendix.

By September 1, 1970, the proposed component I activities had been completed; the resource units had been developed and computer programs were operational.

Resource Units Class Tested - The proposed activities of component II of this research project included the class testing of the resource units and programs developed during component I. Upon notification of approval of this project by USOE, arrangements were made for the principal investigator to teach two pre-calculus classes during the semester beginning September, 1970. As proposed, one of these classes used the project resource materials and one class followed the regular class pattern of lecture, discussion, etc.

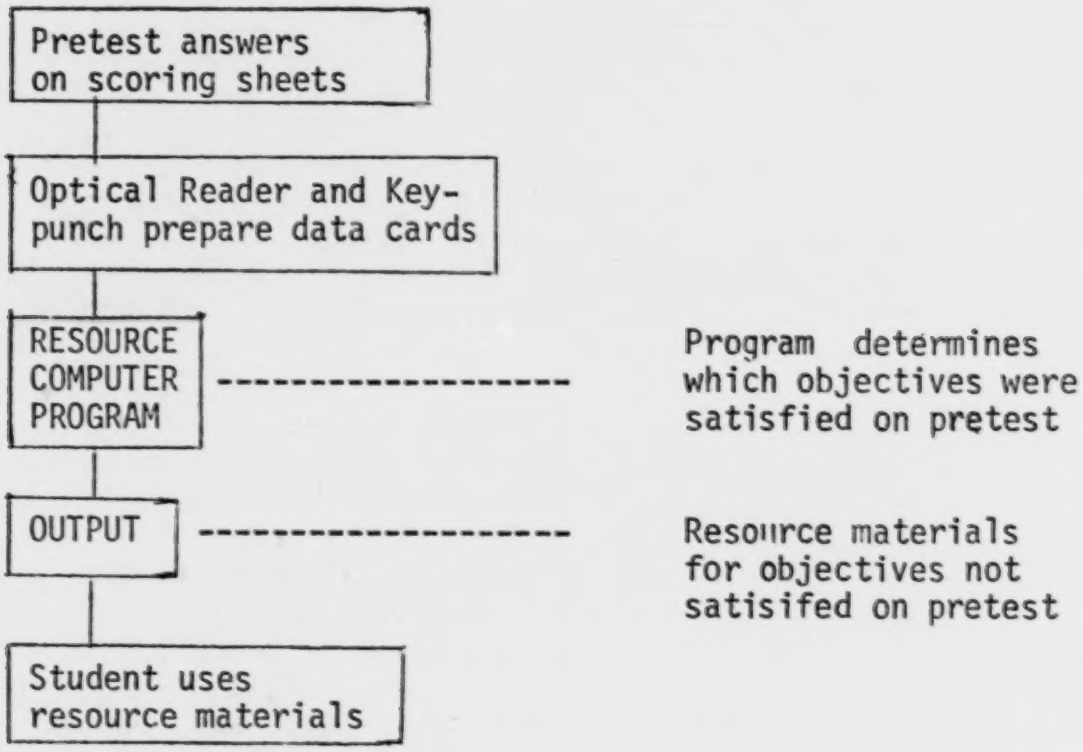
Two resource centers were established for the class using the resource units. The center receiving greatest use was located in the college library in cooperation with the Reserved Book Department. Multiple copies of all materials were available and students could take the materials out of the library overnight. A second resource center was located near the principal investigator's office. This center was used primarily by commuting students who desired to take the materials home but would have difficulty meeting the library time limits.

The pre-calculus class using the resource units met once a week on Tuesday evenings for fourteen weeks. At the first class meeting on Tuesday, September 8, 1970, the Sets, Relations and Functions pretest was administered. The answer sheets were processed immediately and returned to the students later in the class. The output was explained and the students worked on this unit for the first three weeks of the semester. The second unit pretest, Algebra, and the third, Elementary Functions, were administered on one Tuesday and the output returned the next week. The fourth unit pretest, Analytic Geometry, was returned to the student by mail in order to meet the semester schedule. In addition to two unit examinations, both pre-calculus classes took a sixty-eight item multiple-choice final.

A subset of the questions on the four unit pretests was used as a pretest for approximately three hundred students beginning calculus in September, 1970. These pretests results were compared with final grades in calculus to determine if success on the pretests is related to success in calculus.

Evaluation of Resource Units - The Resource Units were evaluated in two ways. First, achievement of the students using the resource units was compared with achievement of the students in the control class using a one-way analysis of variance model applied to the four variables: Unit Test I, Unit Test II, Final Exam and Final Letter Grades. A more subjective evaluation of the units involved observation of student reaction to the units. Extent of student use of materials and student comments on the units were considered in this evaluation.

DIAGRAM OF RESOURCE UNIT OPERATION


Pretest answers on scoring sheets
Optical Reader and Key-punch prepare data cards
RESOURCE COMPUTER PROGRAM
Program determines which objectives were satisfied on pretest
OUTPUT
Resource materials for objectives not satisifed on pretest
Student uses resource materials

Results

A correlation matrix for the control group data is presented in Table I. Table II presents the experimental group correlation matrix. Intercorrelations of achievement variables were approximately the same for both groups. The pretest administered to the control group had a correlation of approximately .35 with each of the achievement variables. In the experimental group considerable variability exists among the correlations of unit pretests with the achievement variables. The algebra pretest correlated best with the achievement measures.

Means and standard deviations for all achievement variables are presented in table III. Table IV presents descriptive data for each of the four Unit Pretests. The relatively large standard deviations for all pretests indicates the extreme variability in student pretest scores. For each unit the scores ranged from lows of 0 to 5% to highs of 90% or better.

Table V presents the analysis of variance for each of the achievement variables. For Unit Test I, Final Exam, and Final Letter Grade variables, the F-ratio was less than one and the hypothesis of no significant difference between the two groups on these achievement measures could not be rejected. For Unit Test II the F-ratio of 5,80 was significant at the 5% level and the hypothesis of no significant difference between the two groups on this achievement measure must be rejected.

The calculus pretest consisting of questions selected from each of the four unit pretests was administered to 335 beginning calculus students and to 28 students in the pre-calculus class which served as the control group for the study. Histograms for these groups are presented in table VI. It will be noted that the mean for the calculus students was 53.1%, whereas, the mean for the pre-calculus students was 23.7% with only a few of the calculus students scoring as low as the mean for the pre-calculus students and only one pre-calculus student scoring as high as the mean of the calculus class. Final grades for the calculus students were correlated with the pretest scores. The correlation was .45, with very few successes (grades of A, B, or C) in calculus for students who scored low on the pretest.

The instructor evaluated the application of the resource units through numerous conferences with student users. Student reaction was generally favorable, especially with students who were deficient on only a few objectives of a given unit. No provision was made for acceleration by the students with few deficiencies. Such an opportunity would have been welcomed by several students. The use of a variety of teaching materials for a single unit of mathematics provided several student reactions. The instructor received many verbal evaluations and comparisons of teaching materials. After using two programmed books for the same topic, some students freely indicated a preference for one book based upon the book's readability or the author's approach to the topic. The anticipated problem of differing mathematical notations did trouble some students but most students seemed to overcome this annoyance. Perhaps these students now see mathematical concepts as being independent of mathematical notation.

Table I

CONTROL GROUP CORRELATION MATRIX

	Pretest	Unit I	Unit II	Final Exam	Letter Grade
Pretest	1.00				
Unit Test I	.34	1.00			
Unit Test II	.38	.68	1.00		
Final Exam	.33	.84	.78	1.00	
Letter Grade	.33	.80	.82	.85	1.00

Table II

EXPERIMENTAL GROUP CORRELATION MATRIX

	Sets	Algb.	Fctns.	Geo.	Unit I	Unit II	Final	L. G.
Sets Pretest	1.00							
Algebra Pretest	.31	1.00						
Functions Pretest	.37	.50	1.00					
Geometry Pretest	.13	.49	.34	1.00				
Unit Test I	.27	.35	.35	.19	1.00			
Unit Test II	.44	.66	.48	.27	.58	1.00		
Final Exam	.44	.70	.52	.50	.64	.73	1.00	
Letter Grades	.38	.62	.53	.42	.70	.83	.89	1.00

Table III

MEANS AND STANDARD DEVIATIONS
FOR ACHIEVEMENT VARIABLES

	Control Group n = 23		Experimental Group n = 21	
	Mean	S. D.	Mean	S. D.
Unit Test I	36.17	7.13	34.95	6.49
Unit Test II	42.13	11.96	33.38	12.11
Final Exam	64.96	17.65	60.91	13.59
Letter Grade	2.26	1.14	2.00	0.89

Table IV

DATA ON RESOURCE UNIT PRETESTS

Pretest	Number of items	Mean percent correct	Standard deviation	Kuder-Richardson For. -21	Ave. Discrim-ination	Ave. difficulty index
Sets	60	52.0	27.5	.96	.66	.48
Algebra	49	44.5	18.5	.87	.44	.55
El. Fctns.	60	36.3	16.7	.88	.40	.64
Geometry	46	34.8	17.0	.85	.38	.65

Table V

ANALYSIS OF VARIANCE

UNIT TEST I

	Sum of Squares	df	Mean Square	F Ratio
Between Groups	163.79	1	163.79	.36
Within Groups	19602.56	42	466.73	
Total	19766.35	43		

UNIT TEST II

	Sum of Squares	df	Mean Square	F Ratio
Between Groups	840.35	1	840.35	5.80
Within Groups	6083.56	42	144.85	
Total	6923.91	43		

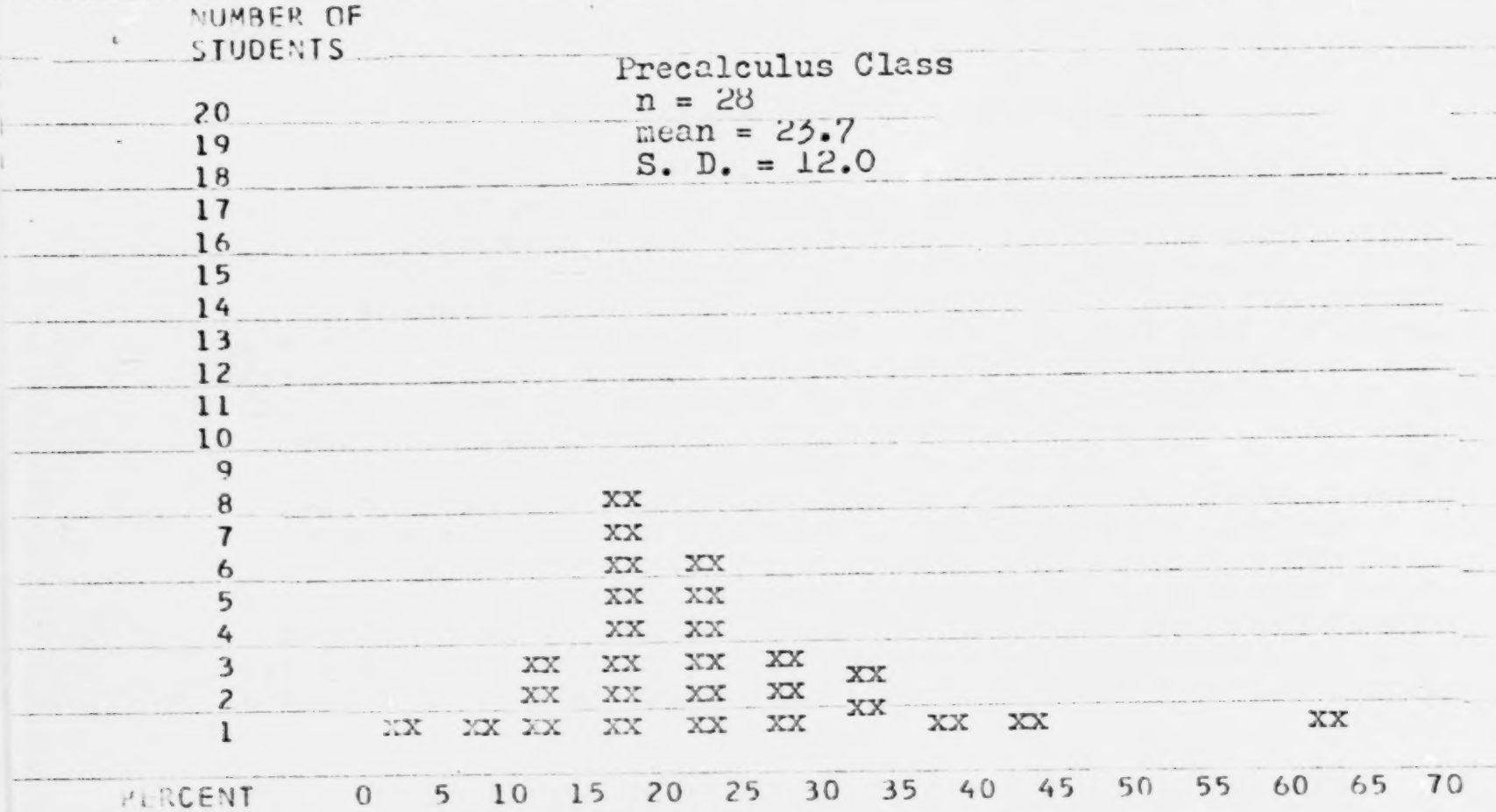
FINAL EXAM

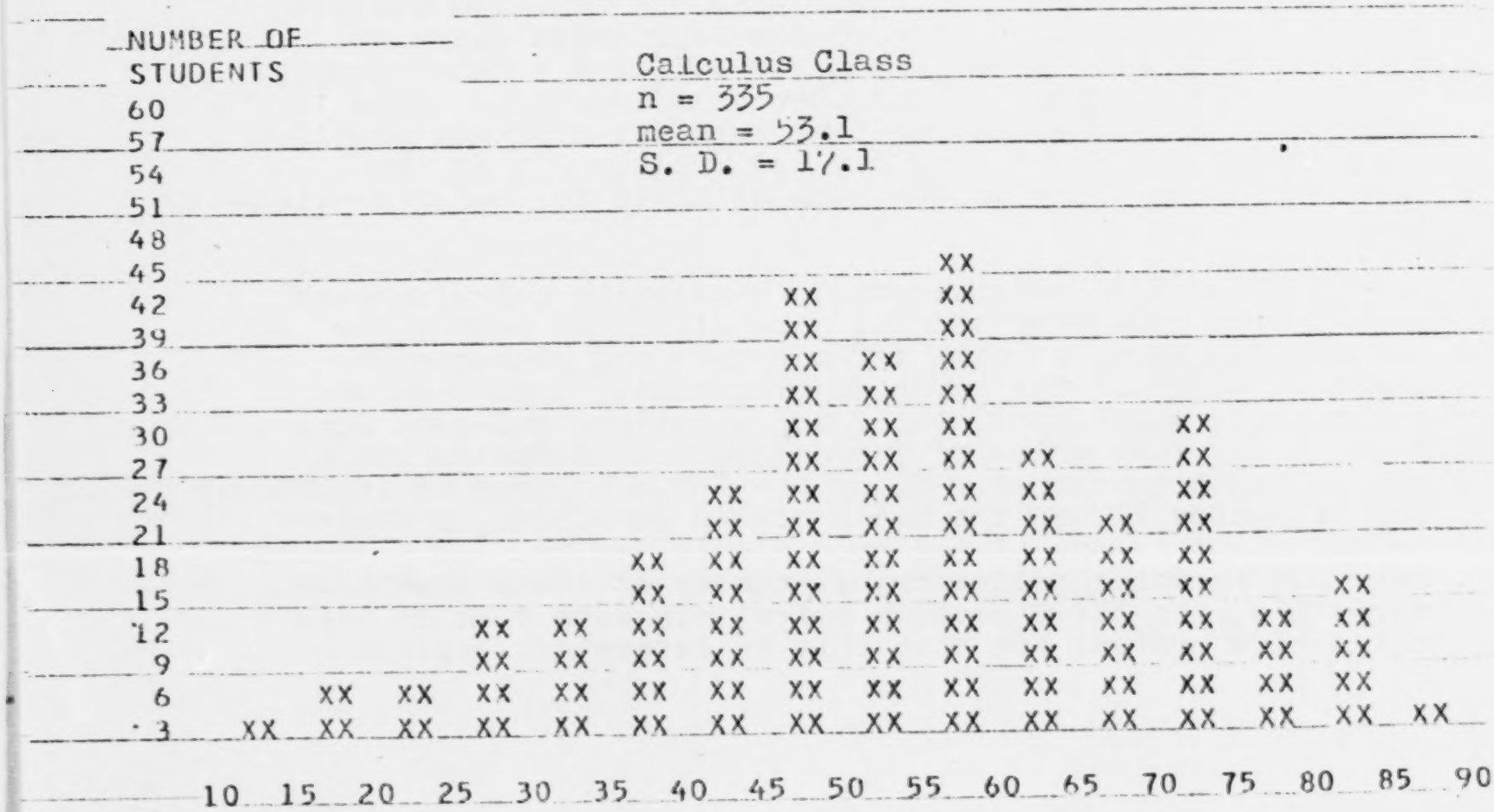
	Sum of Squares	df	Mean Square	F Ratio
Between Groups	180.21	1	180.21	.72
Within Groups	10548.77	42	251.16	
Total	10728.98	43		

FINAL LETTER GRADES

	Sum of Squares	df	Mean Square	F Ratio
Between Groups	7.47	1	7.47	.70
Within Groups	444.35	42	10.58	
Total	451.82	43		

Table VI
HISTOGRAM FOR CALCULUS PRETEST





Conclusions and Recommendations

As noted earlier, the two pre-calculus classes were compared using analysis of variance applied to four achievement variables. For three of these measures, Unit Test I, Final Exam, and Final Letter Grades, the null hypothesis could not be rejected. For one achievement measure, Unit Test II, the control group scored significantly higher than the experimental group. This researcher suspects that if multivariate analysis of variance had been used with this data, especially with a suitable covariate, that the hypothesis of equal mean vectors would not be rejected. Unfortunately the design of the experiment did not include a covariate. The fact that one of the classes was a scheduled evening class raises some question about the value of these achievement comparisons without the use of appropriate covariates. In the light of these factors it is concluded that, in general, the achievement of the control group was not significantly better than the achievement of the experimental group.

Although this experiment provided an opportunity to try the resource units with a large number of students at the same time, it may not have been the best test of the resource unit concept. No provision was made for allowing students to accelerate through the four units since the pretests and unit tests were given at fixed times. Perhaps the best application of computer based resource units is in providing review and remedial work for students enrolled in a course for which they may not satisfy all of the prerequisites. Such is the case with the following application.

With several hundred students entering calculus each year it was necessary to find some efficient method for identifying students with deficiencies and students who might be eligible for advanced placement. A pretest was constructed which consists of ten questions selected from each of the four units of pre-calculus mathematics and a few calculus questions. Using the INFOS program an instructor may determine if, on the basis of ten questions, a student is deficient in one of the pre-calculus units. If a student does not answer a predetermined number of these questions correctly, the INFOS program output suggests that he take the entire pretest for that unit to determine specific

deficiencies and corresponding resource materials for overcoming these deficiencies. Students who are successful with the calculus questions are invited to discuss the possibility of advanced placement with the course instructor. The INFOS output also advises these students to provide the instructor with information on their previous calculus study such as text title, oourse outline or course exams. Times and places for taking the pretests and meeting with instructors may also be included in the output for each student.

A recommended calculus pretest includes the following questions selected from the four unit pretests appended to this report: Algebra pretest questions 10,13,19,24,28,33,36, 38,41,45; Analytic Geometry questions 2,9,10,15,18,21,26,30, 43,45; Sets, Relations and Functions questions 6,20,28,32,36, 40,41,47,50,54; Elementary Functions questions 5,11,17,19,26, 35,41,47,52,54. This revised calculus pretest will be used with beginning calculus students starting in the Fall, 1971.

The resource units have been used by several individuals not enrolled in any mathematics course. A college staff member and a number of students in other disciplines are using the resource units as a means of review and preparation for calculus courses required in their academic programs. The pre-calculus resource units are also being used by some continuing education students to satisfy requirements for a Bachelor of Liberal Studies Degree.

The resource units could also be used in vocational educational programs. A subset of the unit objectives could be selected which would be relevant for a specific vocation or industrial position.

The computer based resource unit approach to teaching pre-college mathematics to post high school students could be used by mathematics departments involved in implementing full opportunity programs for all high school graduates. An effective and efficient means of teaching pre-college mathematics is imperative for colleges participating in such programs. The resource units could be especially useful where student tutors are involved.

Individualizing instruction through the use of computer based resource units need not be limited to the teaching of mathematics. Certainly there is a need in other disciplines

to bring able students of varied backgrounds to the same level of achievement in a most efficient way. Currently, a fifth grade teacher is preparing to use the INFOS program for individualizing language arts instruction and several social science colleagues have expressed an interest in using the program with undergraduate students.

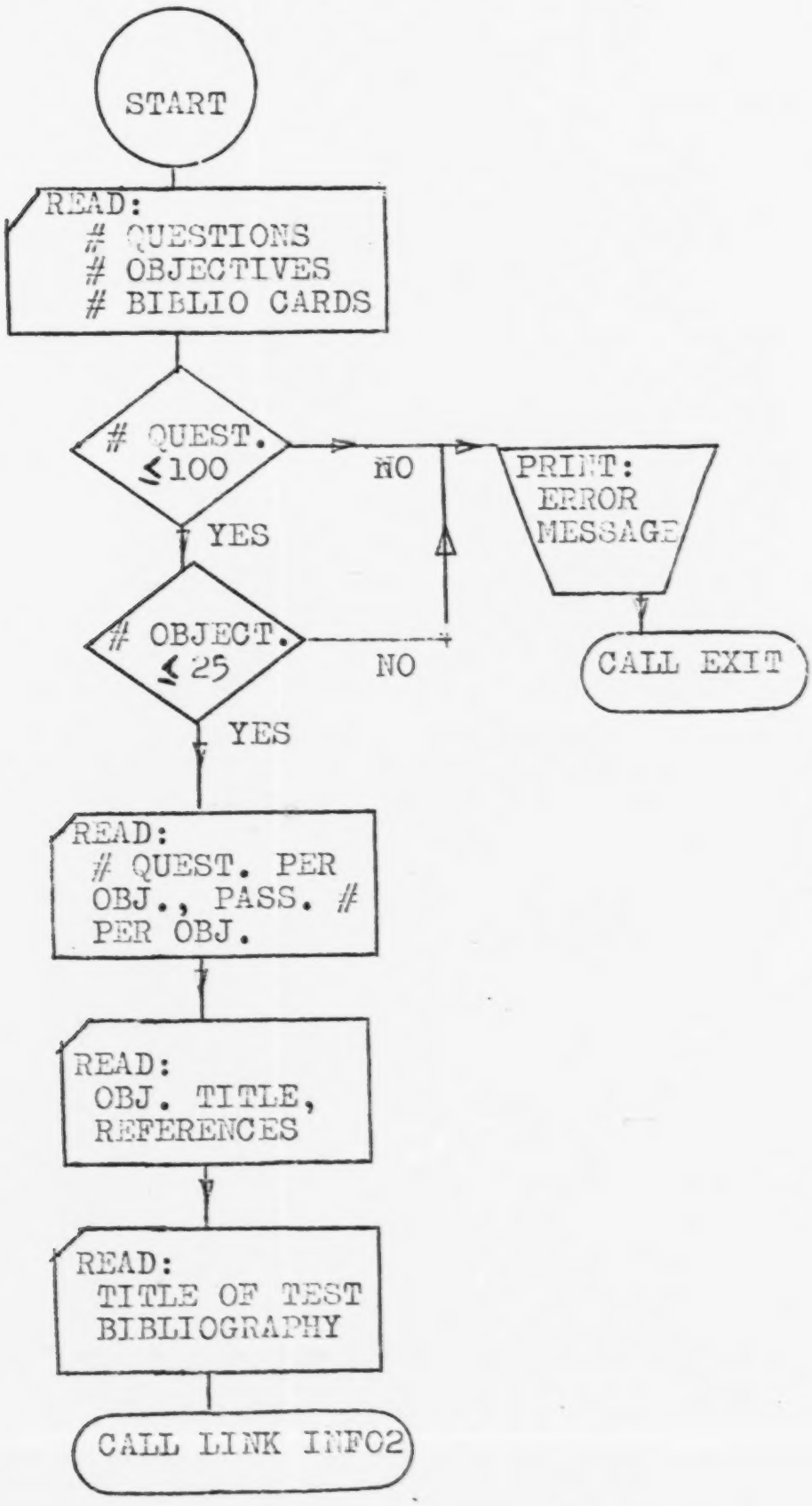
The general program INFOS could be used by instructors in any discipline. To use INFOS instructors must indicate general information about the pretest such as number of questions and the number of objectives represented by the test. For each objective the user must provide a title for the objective, the number of questions for the objective, the question numbers for the objective, the number right which satisfies the objective, and the message to the student if the objective is not satisfied. Since interaction with the computer is by way of scoring sheets and printed output, instructors do not need direct access to a computer but could use any college or commerical computer. A major advantage of this inexpensive use of the computer is the opportunity to focus the experience of several instructors upon the needs of a single student.

A report on this project and a description of the INFOS program will appear soon in the New Programs Department of The Mathematics Teacher. A more extensive article on the project has been submitted to School Science and Mathematics.

APPENDIX

INFOS Flowchart
INFOS FORTRAN Listing
INFO2 Flowchart
INFO2 FORTRAN Listing
Directions to Instructors
Directions to Students
Sets, Relations and Function Unit
    Objectives
    Pretest
    Sample Output
    Program Listing
Algebra Unit
    Objective
    Pretest
    Sample Output
    Program Listing
Elementary Functions Unit
    Objectives
    Pretest
    Sample Output
    Program Listing
Analytic Geometry Unit
    Objectives
    Pretest
    Sample Output
    Program Listing

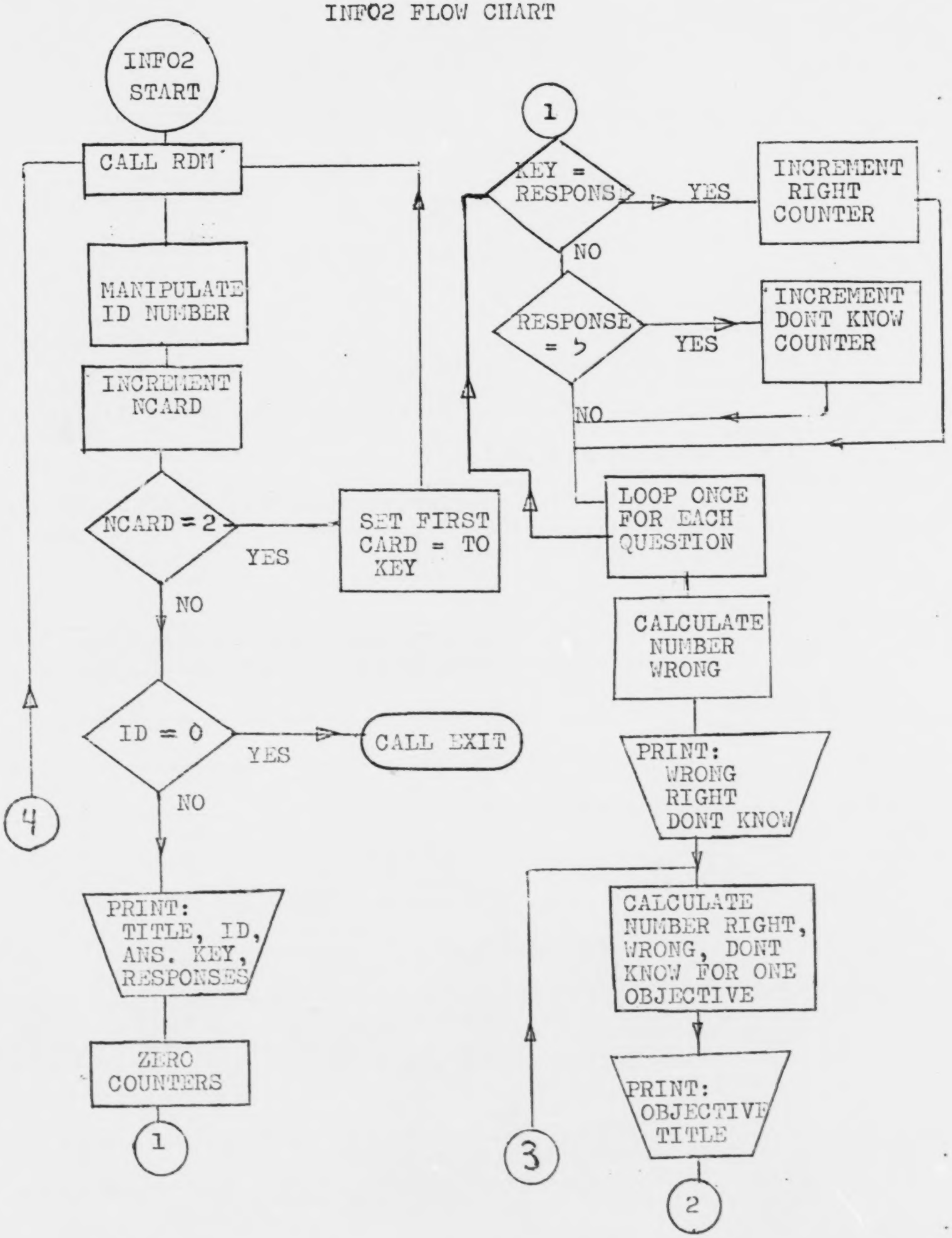
INFOS FLOW CHART



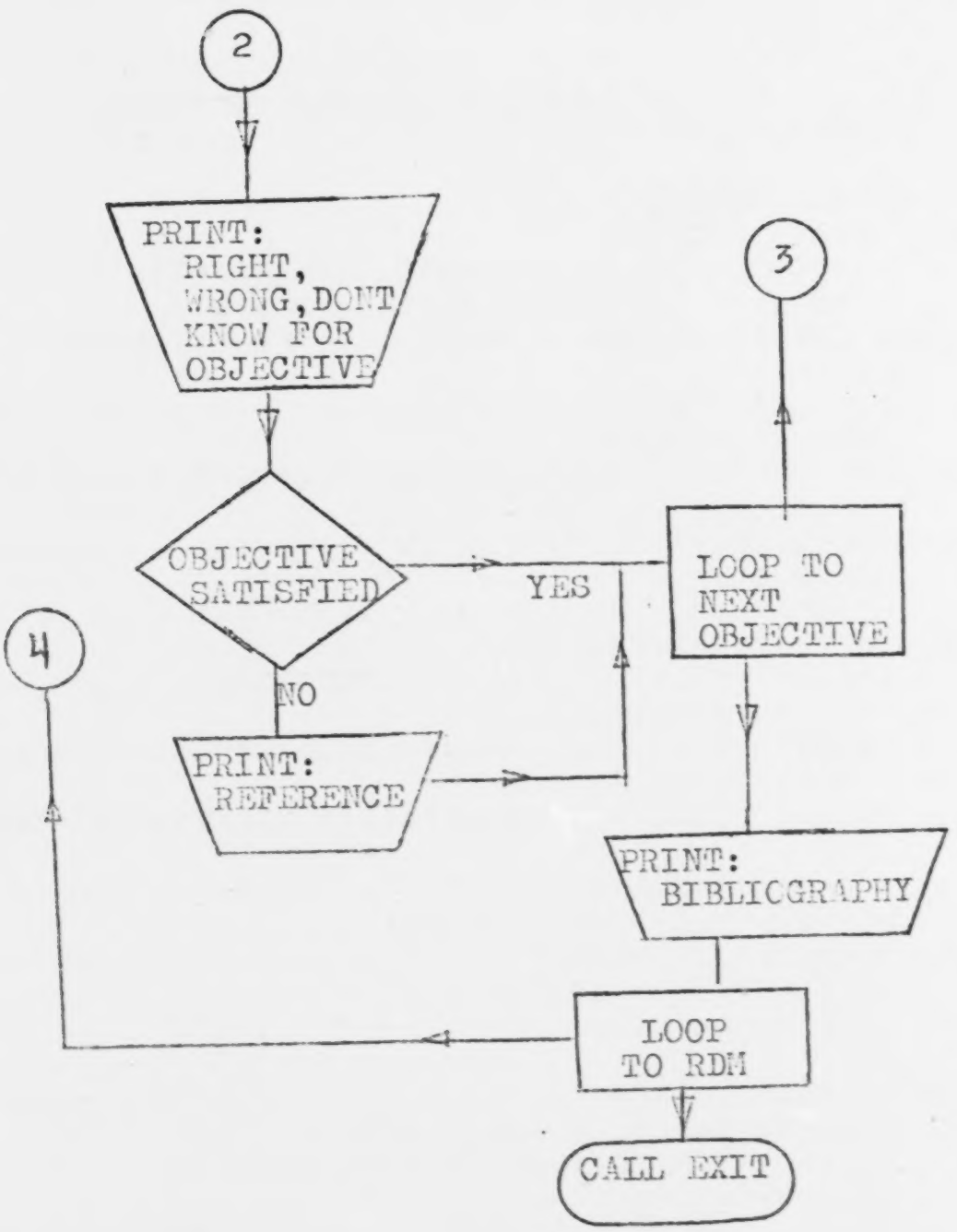
FORTRAN LISTING OF INFOS

```
// JOB    CCC1
// FOR
*ONE WORD INTEGERS
*IOCS(CARD,1403PRINTER)
*LIST SOURCE PROGRAM
      DIMENSION NQPC(25),IPGPC(25),ITQN(25,10 )
      DIMENSION ICBJT(25,40),IREF(25,80),
     TITITL(40),KBIBL(10,40),  N(140),KEY(100),IRIGT(25),IRONG(25),
     TIDNK(25)
      DIMENSION LIST (140,26)
      DIMENSION ID(10)
      COMMON NQ,LIST,KEY,NOBJ,NQPC,IRIGT,IDNK,ITQN,ICBJT,IRONG,IREF,IBIB
     TL,KBIBL,IPGPC,ITITL,ID
      EQUIVALENCE(LIST(1,26),N(1))
      KI=2
      KO=5
    1 FORMAT (3I4)
    3 FORMAT (1X,'ERROR-- NUMBER OF QUESTIONS EXCEEDS 100')
    6 FORMAT (1X,'ERROR-- NUMBER OF OBJECTIVES EXCEEDS 25')
    8 FORMAT (20(I3,1X))
   14 FORMAT (10I5)
   16 FORMAT (40A2)
      READ (KI,1)NQ,NOBJ,IBIBL
      NR=(NQ+1)/2
      IF (NQ-100)4,4,2
    2 WRITE (KO,3)
      CALL EXIT
    4 IF (NOBJ-25)7,7,5
    5 WRITE (KO,6)
      CALL EXIT
    7 READ (KI,8) (NQPC(I),I=1,NOBJ)
      IF (I-20)10,10,9
    9 READ (KI,8)(NQPC(I),I=21,NOBJ)
   10 READ (KI,8)(IPGPC(I),I=1,NOBJ)
      IF (I-20)12,12,11
   11 READ (KI,8)(IPGPC(I),I=21,NOBJ)
   12 DO 15 I=1,NOBJ
      ISRCC=NQPC(I)
      READ (KI,14)(ITQN(I,J),J=1,ISRCC)
      READ (KI,16)(ICBJT(I,JK),JK=1,40)
      READ (KI,16)(IREF(I,II),II=1,40)
      READ (KI,16)(IREF(I,II),II=41,80)
   15 CONTINUE
      READ (KI,16)(ITITL (L),L=1,40)
      DO 36LL=1,IBIBL
   36 READ (KI,16)(KBIBL(LL,L),L=1,40)
      CALL LINK(INFO2)
      END
```

INFO2 FLOW CHART



INFC2 FLOW CHART CONTINUED



FORTRAN LISTING OF INFO2

```
*ONE WORD INTEGERS
*LIST SOURCE PROGRAM
      DIMENSION NQPC(25),IPGPC(25),ITCN(25,10 )
      DIMENSION ICBJT(25,40),IREF(25,80),
     TITITL(40),KBIBL(10,40),  N(140),KEY(100),IRIGT(25),IRONG(25),
     T IDNK(25)
      DIMENSION LIST (140,26)
      DIMENSION ID(10)
      COMMON NQ,LIST,KEY,NORJ,NQPC,IRIGT,IDNK,ITQN,ICBJT,IRONG,IREF,IBIB
     TL,KBIBL,IPGPC,ITITL,ID
      EQUIVALENCE(LIST(1,26),N(1))
      NCARD=1
      KC=5
   23 FORMAT (1H1,20X,40A2,/)
   27 FORMAT (5X,'ANS. KEY',2X,100I1)
   28 FORMAT (5X,9I1,1X,100I1)
   35 FORMAT (/10X,I4,' RIGHT',I4,' WRONG',I4,' DONT KNOW')
   44 FORMAT (/25X,40A2)
   45 FORMAT (10X,I3,' RIGHT',I3,' WRONG',I3,' DONT KNOW',3X,'OUT OF 'I3
     T)
   47 FORMAT (10X,55A2/25A2)
   50 FORMAT (1H1,////30X,40A2)
   55 FORMAT (/20X,40A2)
      CALL RDM (N,M1,M2,M3,0)
   22 CALL RDM(N,M1,M2,M3,1)
      L=1
      ID(L)=M1/100
      ID(L+1)=(M1-(ID(1)*100))/10
      ID(L+2)=M1-(ID(1)*100)-(ID(2)*10)
      ID(L+3)=M2/10
      ID (L+4)=M2-(ID(4)*10)
      ID(L+5)=M3/1000
      ID(L+6)=(M3-(ID(6)*1000))/100
      ID(L+7)=((M3-ID(6)*1000)-(ID(7)*100))/10
      ID(L+8)=M3-(ID(6)*1000)-(ID(7)*100)-(ID(8)*10)
      NCARD=NCARD+1
      IF (NCARD-2)26,24,26
   24 DO 25 L=1,NQ
   25 KEY(L)=LIST (L,26)
      GO TO 22
   26 IF (M3)30,500,30
   30 WRITE (KC,23)(ITITL(L),L=1,40)
      WRITE (KC,27)(KEY(L),L=1,NQ)
      WRITE (KC,28)(ID(L),L=1,9),(LIST(LL,26),LL=1,NQ)
```

FORTRAN LISTING OF INFO2 CONTINUED

```
      KDNK=C
      KRIGT=C
      DC 34 K=1,NC
      IF (KEY(K)-LIST(K,26))32,33,32
   33 KRIGT=KRIGT+1
      GC TC 34
   32 IF (LIST(K,26)-5)34,31,34
   31 KDNK=KDNK+1
   34 CONTINUE
      KRONG=NC-KRIGT-KDNK
      WRITE (KO,35)KRIGT,KRONG,KDNK
      DO 48 I=1,NOBJ

      ISRCC=NGPO(I)
      IRIGT(I)=0
      IDNK(I)=0
      DO 43 J=1,ISRCC
      IITQN=ITQN(I,J)
      LIST(J,I)=LIST(IITQN,26)
      IF (LIST(J,I)-KEY(IITQN))41,40,41
   40 IRIGT(I)=IRIGT(I)+1
   41 IF (LIST(J,I)-5)43,42,43
   42 IDNK(I)=IDNK(I)+1
   43 CONTINUE
      WRITE (KC,44)(ICBJT(I,JK),JK=1,40)
      IRONG(I)=NGPC(I)-IRIGT(I)-IDNK(I)
      WRITE (KO,45)IRIGT(I),IRONG(I),IDNK(I),NGPO(I)
      IF (IPGPC(I)-IRIGT(I))48,48,46
   46 WRITE (KO,47)(IREF(I,II),II=1,80)
   48 CONTINUE
      DO 49 LL=1,IBIBL
      IF (LL-1)56,52,56
   52 WRITE (KC,5C)(KBIBL(LL,L),L=1,4C)
      GC TC 49
   56 WRITE (KO,55)(KBIBL(LL,L),L=1,40)
   49 CONTINUE
      GC TC 22
  5CC CALL EXIT
      END
```

Directions To Instructors

Development of the Resource Units

During the summer, 1970, four experienced calculus teachers determined objectives which should be satisfied by students entering calculus. Four units of pre-calculus mathematics were considered: Sets, Relations and Functions; Algebra; Elementary Functions and Analytic Geometry. The teachers then wrote multiple-choice items to test each objective. Usually, four test items were written for each objective.

A Fortran IV computer program was then written which would take the student's answers to the test items for a unit as input, determine which objectives were not satisfied by the student and provide a printout of how successful the student was on each objective, together with suggested teaching materials for those objectives which were not satisfied.

A common program INFOS is used by all four units. This program allows for varing the number of objectives, the number of questions per objective, the number correct for success on an objective, and the printout of teaching materials.

During the Fall, 1970, semester all four units were used with a class of twenty-five pre-calculus students. Following this experience minor revisions were made in each unit.

Using the Resource Units

The Fortran program, INFOS, should be adaptable for use with any computing system. The program uses subroutines to translate card input from an IBM 1230 Optical Reader Keypunch system which produces one card per student with responses punched two per column. Some adjustments may be necessary to make the program compatible with local input-output devices. Since each of the four unit programs is essentially a data deck for the INFOS program, a user may conveniently make changes, deletions or additions to adapt each unit for local use. It should be noted that questions related to a given objective need not be consecutive and a given question may be used for more than one objective.

When administering a pretest an instructor should (1) be sure students use the appropriate pencil (usually a #2 or equivalent), (2) discuss the purpose of the pretest so students will not guess but select choice five when appropriate, (3) be sure student identification numbers appear on the answer sheets.

Fifty minutes should be sufficient for most students to complete each of the unit pretests. Experience has shown that placing several copies of each of the teaching materials on reserve in the library will be sufficient for student access, especially if they may be taken out of the library overnight.

Directions to the Students

The attached multiple-choice pretest covers one unit of pre-calculus mathematics. The purpose of this test is to identify your strengths and weaknesses for this unit and to recommend some teaching materials covering any weaknesses which are identified.

You should indicate your response to each question on the scoring sheet using a number two pencil. Do not guess. If a question refers to some topic which you have not studied or do not remember, please respond with choice five "I do not know." It is important that your social security number or some other identification number is entered in the space provided at the top of the answer sheet.

Your answer sheet will be used as data for a computer program. This program determines which questions you missed, what your weaknesses are in this unit and what materials you should study. A printout will be returned to you which details the program treatment of your answers. For each objective which you did not satisfy, several references will be printed. You are advised to use these references in the order in which they appear on the printout. Your instructor will tell you where these materials are available.

Objectives for Sets, Relations and Functions Unit

I. Sets

1. To understand the notation and uses of the undefined terms: set and element. (Pretest questions 1,2,4)

2. To know the definitions of: universal set, empty set, disjoint sets. (Pretest questions 3,5,10,11)

3. To know the definitions and notations of the set relations: subset ($\subseteq$), proper subset ($\subset$), equal sets (=), one to one correspondence ($\leftrightarrow$); and to be able to apply these definitions in specific examples. (Pretest questions 1, 2, 6, 8, 9)

4. To know the number of subsets in a given set. (Pretest question 12)

5. To know the definitions and notations of the set operations: union ($\cup$), interesection ($\cap$), complement ('); and to be able to perform these operations on specific sets. (Pretest questions 13, 14, 15, 16)

6. To know the basic properties of the set operations: union, intersection and complement. (Pretest questions 17, 18, 19, 20, 21, 22)

7. To understand and be able to apply the definition of set Cartesian product (X). (Pretest questions 23, 24, 25, 26, 27)

II. Relations

8. To know that a binary relation from set A to set B is a subset of the Cartesian product of A and B. (Pretest question 28)

9. To be able to list the set of ordered pairs in a described relation. (Pretest questions 29, 30)

10. To be able to describe the relationship represented by a set of ordered pairs. (Pretest question 31)

11. To know the reflexive, symmetric and transitive properties of relations, (Pretest questions 32, 33, 34, 35)

III. Function

12. To know the definition of the term function, including the ordered pair definition. (Pretest questions 36, 37, 38, 39)

13. To be able to determine the range and domain of a given function. (Pretest questions 40, 41, 42, 43, 44)

14. For a given function, to be able to determine the value of the range which corresponds to a specific value of the domain. (Pretest questions 45, 46, 47)

15. To recognize the sketches of the constant functions, identity functions and absolute value function. (Pretest questions 48, 49, 50)

16. To be able to identify functions for which inverse functions exist. (Pretest question 51, 53)

17. To be able to identify the inverse of a given function. (Pretest question 52)

18. To be able to identify the composite function $g(f(x))$, given the two functions $f(x)$ and $g(x)$. (Pretest questions 54, 55)

19. To be able to indicate the range and domain of the composite functions $g(f(x))$. (Pretest questions 56, 57)

20. To be able to identify the domain of the sum, product and quotient of two functions.

Sets, Relations and Functions Pretest

1. If set A = {r, s, t} then:
   (1) r is an element of A
   (2) r is a subset of A
   (3) A is a subset of r
   (4) A is an element of r
   (5) I do not know

2. If A = {x| x is an odd number less than 10} then it follows that:
   (1) 9 is a subset of A.
   (2) $9 \not\subset A$
   (3) $9 \notin A$
   (4) 9 is a proper subset of A
   (5) I do not know.

3. In any discussion the set which contains all members of any set which we are considering is called:
   (1) a subset
   (2) the universal set
   (3) the empty set
   (4) disjoint
   (5) I do not know

4. If $a \in X$, $b \notin Z$ and X is a subset of Z, which of the following is necessarily true?
   (1) $a \notin Z$
   (2) $b \notin X$
   (3) a is a subset of Z
   (4) a = b
   (5) I do not know

5. Two sets A and B are disjoint if and only if which of the following is true.
   (1) A and B have no elements in common
   (2) A is a subset of B
   (3) A = B
   (4) B is a subset of A
   (5) I do not know

6. If A = {1,3,5,7,9}, B = {3,5,7}, and C = {3,7,9}, which of the following is true?
   (1) A is a subset of B
   (2) B is a subset of C
   (3) C is a subset of A
   (4) B = C
   (5) I do not know

7. If the universal set U = {a, e, i, o, u} and A = {e, i}, then A' (the complement of A) equals:
   (1) {e,i}
   (2) {a,e,i,o,u}
   (3) {a,o,u}
   (4) none of the first three choices
   (5) I do not know

8. If A = {x,y,z} which of the following is <u>not</u> a proper subset of A?
   (1) {x,y}
   (2) {y}
   (3) $\emptyset$
   (4) {x,y,z}
   (5) I do not know

9. If there exists a one-to-one correspondence between the finite sets A and B then it follows that:
   (1) A = B
   (2) A is a subset of B
   (3) A is a proper subset of B
   (4) A has the same cardinal number as B
   (5) I do not know

10. Which of the first three choices below does <u>not</u> represent the empty set?

   (1) A = {x | x is an odd natural number which is divisible by 4}
   (2) B = {-1,0,1}
   (3) $\emptyset$
   (4) All of the first three choices represent the empty set
   (5) I do not know

11. (A) The empty set is a subset of every set.
   (B) The empty set is an element of every set.
   (C) The complement of the empty set is the universal set.
   Which of the statements above is false?

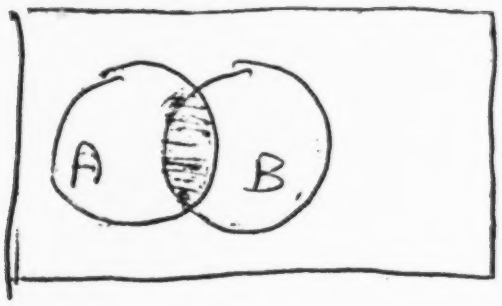
   (1) A is false
   (2) B is false
   (3) C is false
   (4) Statements A,B and C are all true
   (5) I do not know

12. If set A contains n elements then the number of subsets of A is:
   (1) n
   (2) n^2
   (3) 2^n
   (4) none of the above
   (5) I do not know.

13. If A = {3,5,7} and B = {5,7 9,11} then $A \cap B$ equals:

   (1) {3,5,7,9,11}
   (2) {9,11}
   (3) {5,7}
   (4) none of the first three choices
   (5) I do not know.

14. In the Venn diagram at the right the shaded region represents

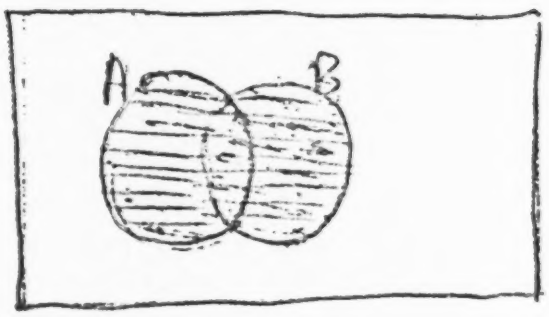


(1) $A \cap B$
(2) $A \cup B$
(3) A'
(4) none of the first three choices
(5) I do not know.

15. If $A = \{3,5,7\}$ and $B = \{5,7,9,11\}$ then $A \cup B$ equals:

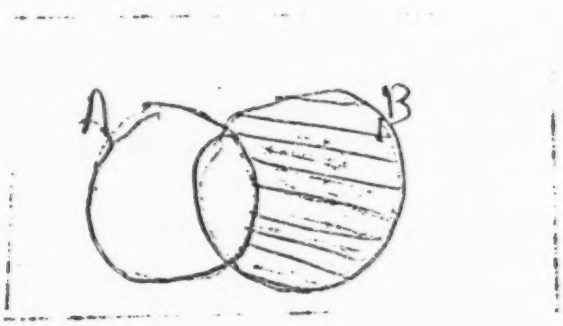
(1) $\{3,5,7,9,11\}$
(2) $\{5,7\}$
(3) $\{3,9,11\}$
(4) none of the first three choices
(5) I do not know.

16. In the diagram at the right the shaded region represents:



(1) $A \cup B$
(2) $A \cap B$
(3) $A' \cup B'$
(4) none of the first three choices
(5) I do not know.

17. In the diagram at the right the shaded region represents:



(1) $A \cap B'$
(2) $A' \cap B$
(3) $A \cap B$
(4) none of the first three choices
(5) I do not know.

18. $A \cap A'$ equals

(1) A
(2) the universal set
(3) the empty set
(4) none of the first three choices
(5) I do not know.

19. $A \cup (B \cup C)$ equals

(1) $A \cap (B \cap C)$
(2) $(A \cup B) \cap C$
(3) $(A \cap B) \cup C$
(4) $(A \cup B) \cup C$
(5) I do not know.

20. $A \cup (B \cap C)$ equals

(1) $(A \cap B) \cup (A \cap C)$
(2) $(A \cup B) \cap (A \cup C)$
(3) $A \cap (B \cup C)$
(4) $(A \cup B) \cap C$
(5) I do not know

21. $(A \cup B)'$ equals

(1) $A' \cup B'$
(2) $A' \cap B'$
(3) $(A \cap B)'$
(4) none of the first three choices
(5) I do not know

22. (1) $A \cup \emptyset = \emptyset$
(2) $(B \cup \emptyset)' = B$

Which of the statements above (is) are true if A and B are any sets?

(1) Both 1 and 2
(2) 1 only
(3) 2 only
(4) neither 1 nor 2
(5) I do not know

23. If $A = \{x,y\}$ and $B = \{a,b,c\}$, which of the following belongs to $A \times B$?

(1) (a,x)
(2) (x,x)
(3) (y,a)
(4) none of the first three choices
(5) I do not know

24. If $A = \{1,2\}$ and $B = \{2,3\}$ then the Cartesian product of A and B ($A \times B$) equals:

(1) $\{(1,2), (2,1), (2,3), (3,2)\}$
(2) $\{(1,2), (1,3), (1,1), (2,2)\}$
(3) $\{(1,2), (1,3), (2,2), (2,3)\}$
(4) none of the above
(5) I do not know

25. (a) $A \times B = B \times A$
(b) $A \times B$ is a subset of $B \times A$
(c) A is a subset of $B \times A$

Which of the above three statements is true?

(1) (a)
(2) (b)
(3) (c)
(4) none of the statements are true
(5) I do not know

26. If A X B = {(5,4), (4,2), (5,3), (4,1)}then A equals:
(1) {5}
(2) {4,5}
(3) {3,4,5}
(4) {1,2,3,4}
(5) I do not know

27. If A,B and C are sets then AX(B ∪ C) equals:
(1) A X (B ∩ C)
(2) (A X B) ∪ C
(3) (A X B) ∪ (A X C)
(4) (A ∪ B) X (A ∪ C)
(5) I do not know

28. A binary relation from A to B is a set of ordered pairs which is a subset of A X B. Which of the following is a binary relation from A to B if A = {5,6} and B = {6,7}?
(1) { (5,7), (6,7) }.
(2) { (5,5) (6,6) }.
(3) { (7,6), (6,6) }.
(4) none of these
(5) I do not know

29. If A = {1,2,3} and B = {2,3} then the binary relation R = { (a b) | a ∈A, b ∈B and a = b} equals:
(1) { (2,2), (3,3) }
(2) { (1,2),(1,3), (2,3) }
(3) { (1,2), (2,2), (2,3) }
(4) { (2,1), (2,2), (3,2) }
(5) I do not know

30. If A = {1,2,3} and B = {2,3,4} then the binary relation R = { (x,y) | x ∈A, y ∈ B and 2x = y} equals:
(1) { (2,2), (3,3) }
(2) { (1,2), (2,3), (3,4) }
(3) { (1,2), (2,4) }
(4) none of these
(5) I do not know

31. If A equals the set of real numbers and B equals the set of real numbers then the relation R = { (x,y) | x ∈ A, y ∈B and $x^2 + y^2 = 1$ } represents:
(1) point
(2) line
(3) circle
(4) ellipse
(5) I do not know

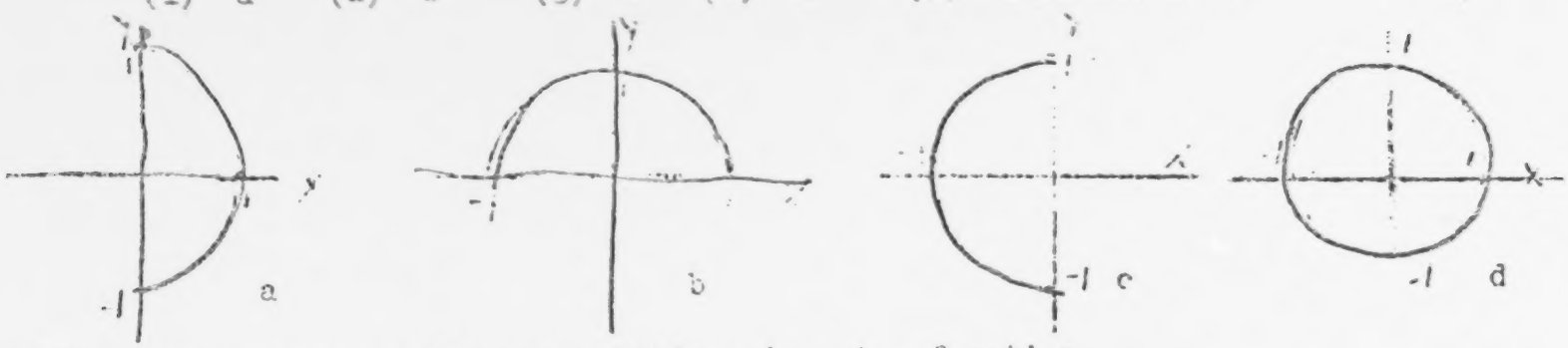
32. The relation "less than" is:
(1) reflexive only
(2) symmetric only
(3) transitive only
(4) reflexive and symmetric
(5) I do not know

33. The relation "subset" is:
(1) reflexive only
(2) reflexive and symmetric
(3) reflexive and transitive
(4) symmetric and transitive
(5) I do not know

34. The relation "brother" is:
(1) reflexive
(2) symmetric
(3) transitive
(4) none of the above
(5) I do not know

35. An equivalence relation is a relation which is:
(1) reflexive only
(2) reflexive and symmetric only
(3) symmetric and transitive only
(4) reflexive, symmetric and transitive
(5) I do not know

36. Which of the relations sketched below is a function:
(1) a (2) b (3) c (4) d (5) I do not know



37. Which of the following relations is not a function.
(1) $R = \{ (x,y) \mid x,y \text{ are real numbers and } y = 5x \}$
(2) $R = \{ (x,y) \mid x,y \text{ are real numbers and } y = x^3 - 1 \}$
(3) $R = \{ (x,y) \mid x,y \text{ are positive real numbers and } xy = 1 \}$
(4) $R = \{ (x,y) \mid x,y \text{ are real numbers and } y^2 = x \}$
(5) I do not know

38. Which of the following relations is <u>not</u> a function.
(1) R = { (1,1), (2,2), (3,3) }
(2) R = { (1,1), (1,2), (1,3) }
(3) R = { (1,1), (2,1), (3,1) }
(4) R = { (1,2), (2,1), (3,3) }
(5) I do not know

39. A function is a relation which assigns to each element in its domain:
(1) Exactly one element in its domain
(2) Exactly one element in its range
(3) One or more elements in its range.
(4) One or more elements in its domain.
(5) I do not know

40. What is the range of the function $f(x) = x^2$?
(1) all real numbers
(2) all positive real numbers
(3) all positive real numbers and zero
(4) none of these
(5) I do not know

41. What is the domain of the function $f(x) = -\sqrt{x}$
(1) all positive real numbers
(2) all positive real numbers and zero
(3) all real numbers
(4) all negative real numbers
(5) I do not know

42. What is the domain of the function $f(x) = \sqrt{9-x^2} = \sqrt{9-x^2}$
(1) $0 \leq x \leq 3$

(2) $0 < x < 3$

(3) $-3 < x < 3$

(4) $-3 \leq x \leq 3$

(5) I do not know

43. What is the domain of the function R = { (1,3), (2,4), (3,5) }
(1) {1,2,3 }
(2) {3,4,5 }
(3) {1,2,3,4,5}
(4) {3}
(5) I do not know

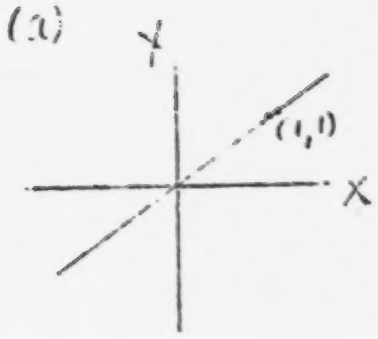
44. What is the range of the function R = { (1,3), (2,4), (3,5) }
(1) {1,2,3}
(2) {3,4,5}
(3) {1.2,3,4,5}
(4) {3}
(5) I do not know

45. If $f(x) = x^2 + 1$ then f(1) equals:
(1) 0 (2) 2 (3) 1 (4) -1 (5) I do not know

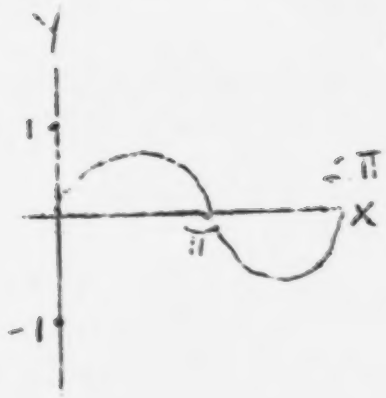
46. If $g(x) = x^2 + 3x$ then g(b) equals:
(1) $b^2 + 3b$ (2) b (3) b^2 (4) $3b^3$ (5) I do not know

47. If $f(x) = x^2$ then $f(x + 2) - f(x)$ equals:
(1) f(x)
(2) $x^2 + 4x + 4$
(3) $4x + 4$
(4) f(2)
(5) I do not know

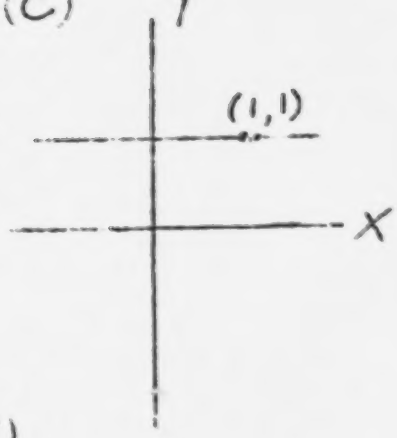
(a)



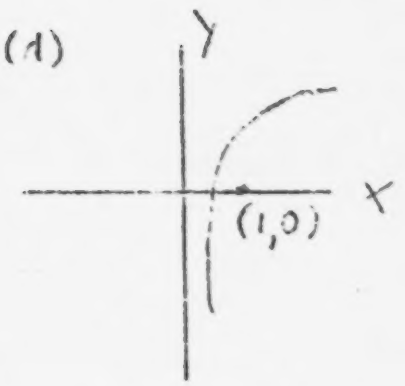
b.



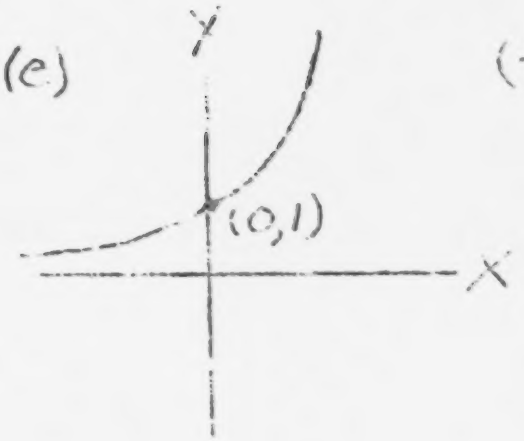
(c)



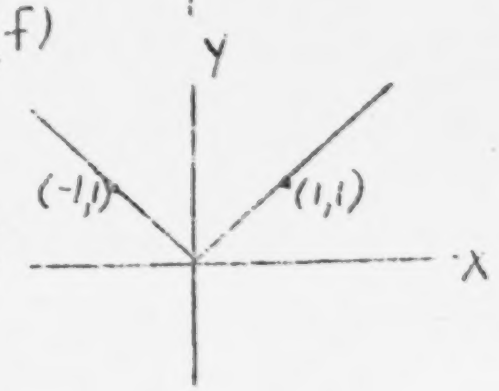
(d)



(e)



(f)



48. The sketch above which represents the identity function y = x is:
(1) f (2) a (3) c (4) d (5) I do not know

49. The sketch above which represents the constant function y = 1 is:
(1) f (2) a (3) c (4) e (5) I do not know

50. The sketch above which represents the function y = |x| is:
(1) c (2) e (3) f (4) a (5) I do not know

51. Which sketch above represents a function which has an inverse for the indicated domain.
(1) a (2) b (3) c (4) f (5) I do not know

52. Which sketch above represents the inverse of the function represented in sketch (d)
(1) b (2) c (3) e (4) f (5) I do not know

53. For which of the following domains would the function $y = x^2$ have an inverse?
(1) $-\infty < x < \infty$
(2) $-1 < x < 1$ only
(3) $0 < x < \infty$ only
(4) none of the above
(5) I do not know

54. If $f(x) = 2x + 1$ and $g(x) = x^2 - 3x$ then $g(f(x))$ equals:
(1) $(2x + 1)^2$
(2) $2(x^2 - 3x) + 1$
(3) $(2x + 1)^2 - 3(2x + 1)$
(4) $(x^2 - 3x) \cdot (2x + 1)$
(5) I do not know.

55. If $g(x)$ and $f(x)$ are two functions defined on the real numbers which of the four statements below may not be true?
(1) $f(x) + g(x) = g(x) + f(x)$
(2) $f(x) \cdot g(x) = g(x) \cdot f(x)$
(3) $f(g(x)) = g(f(x))$
(4) $f(x)(g(x) - f(x)) = f(x)\, g(x) - (f(x))^2$
(5) I do not know

56. If $g(x) = \sqrt{x}$ and $f(x) = 4 - x^2$ then the domain of $g(f(x))$ equals
(1) all real numbers
(2) $x \geq 0$
(3) $-2 \leq x \leq 2$
(4) $0 \leq x \leq 2$
(5) I do not know

57. If $g(x) = \sqrt{x}$ and $f(x) = 4 - x^2$ then the range of $g(f(x))$ equals:
(1) all real numbers
(2) $x \geq 0$
(3) $-2 \leq x \leq 2$
(4) $0 \leq x \leq 2$
(5) I do not know

58. If the function $f(x)$ has domain $-5 < x < 5$ and the function $g(x)$ has domain $3 < x < 7$ then the domain of $f(x) + g(x)$ is:
(1) $-5 < x < 5$
(2) $-5 < x < 7$
(3) $3 < x < 7$
(4) $3 < x < 5$
(5) I do not know

59. If $f(x) = \sqrt{x}$ and $g(x) = x^3$ then the domain of $g(x) \cdot f(x)$ is:
(1) $-\infty < x < \infty$
(2) $0 < x < \infty$
(3) $0 \leq x < \infty$
(4) none of the above
(5) I do not know

60. If $f(x) = x - 2$ and $g(x) = x - 3$ then the domain of $\frac{f(x)}{g(x)}$ would be all real numbers except:
(1) $x = 2$
(2) $x = 3$
(3) $x = 2,3$
(4) $x = 6$
(5) I do not know

**********SETS UNIT PRE-TEST AUG 1970**********

STUDENT NUMBER 055364781
ANS. KEY 132113344223311123422433423113333342422324122132331333334432
055364781 132113344223311153445333124113555555555155335145555555555555

25 RIGHT 8 WRONG 27 DONT KNOW

ELEMENT, SUBSET, UNIVERSAL SET
10 RIGHT 0 WRONG 0 DONT KNOW OUT OF 10

NUMBER OF SUBSETS
2 RIGHT 0 WRONG 0 DONT KNOW OUT OF 2

DEFINITION OF UNION AND INTERSECTION
4 RIGHT 0 WRONG 0 DONT KNOW OUT OF 4

PROPERTIES OF UNION AND INTERSECTION
2 RIGHT 2 WRONG 2 DONT KNOW OUT OF 6
MC FADDEN P55-76 SELBY P40-46

CARTESIAN PRODUCT
3 RIGHT 2 WRONG 0 DONT KNOW OUT OF 5

DEFINITION OF RELATION
3 RIGHT 0 WRONG 1 DONT KNOW OUT OF 4

REFLEXIVE, SYMETRIC, TRANSITIVE PROPERTIES
0 RIGHT 0 WRONG 4 DONT KNOW OUT OF 4
SELBY P156-160 BAUM P68A-74B

DEFINITION OF FUNCTION
0 RIGHT 0 WRONG 4 DONT KNOW OUT OF 4
MC FADDEN(S-R) P187-197 HOWES P76-103(F-R) BAUM P521A-515B

RANGE AND DOMAIN
0 RIGHT 3 WRONG 2 DONT KNOW OUT OF 5
MC FADDEN (S-R)P197-217 BAUM 515B-527A HOWES P104-124(F-R)

FUNCTIONAL VALUES
1 RIGHT 1 WRONG 1 DONT KNOW OUT OF 3
MC FADDEN(S-R)P217-219 BAUM527A-540A HOWES(F-R)P82-86

SKETCH OF IDENTITY FUNCTION
0 RIGHT 2 WRONG 1 DONT KNOW OUT OF 3

MC FADDEN (S-R) P235-242

SKETCH OF THE CONSTANT FUNCTION

0 RIGHT 0 WRONG 1 DONT KNOW OUT OF 1

HOWES(F-R) P219-224

SKETCH OF ABSOLUTE VALUE

0 RIGHT 0 WRONG 1 DONT KNOW OUT OF 1

MC FADDEN (S-R) P235-242

INVERSE FUNCTION

0 RIGHT 0 WRONG 1 DONT KNOW OUT OF 1

MC FADDEN(S-R) P243-255 HOWES(F-R) &78-92,460-464

COMPOSITE FUNCTION

0 RIGHT 0 WRONG 4 DONT KNOW OUT OF 4

MC FADDEN (S-R) P219-230 HOWES P124-146(F-R) SELBY 251-256

SUMS, PRODUCTS, QUOTIENTS OF FUNCTIONS

0 RIGHT 0 WRONG 3 DONT KNOW OUT OF 3

HOWES P124-134 SELBY P234-247

**********SETS UNIT BIBLIOGRAPHY**********

MCFADDEN, MOORE, AND SMITH - SETS, RELATIONS, AND FUNCTIONS

HOWES, VERNON E. - PRE-CALCULUS MATHEMATICS,FUNCTIONS, AND RELATIONS(F-R)

BAUM AND DOBYNS - THE STRUCTURE OF THE REAL NUMBER SYSTEM

SELBY AND SWEET - SETS, RELATIONS AND FUNCTIONS

SETS, RELATIONS, AND FUNCTIONS INPUT FOR INFOS

```
// JOB    CCC1
// XEQ INFOS
  60  16   5
 11   1   4   6   5   4   4   4   5   3   3   1   1   1   4   3
  8   1   3   4   3   3   3   3   3   2   2   1   1   1   3   2


    1    2    3    4    5    6    7    8    9   10   11
ELEMENT, SUBSET, UNIVERSAL SET
MC FADDEN (S-R)P1-39    SELBY P1-22    HOWES P1-33    BAUM 2A-44A

   12
NUMBER OF SUBSETS
SELBY AND SWEET P22-24

   13   14   15   16
DEFINITION OF UNION AND INTERSECTION
MC FADDEN P39-57    BAUM 46A-59A

   17   18   19   20   21   22
PROPERTIES OF UNION  AND INTERSECTION
MC FADDEN P55-76    SELBY P40-46

   23   24   25   26   27
CARTESIAN PRODUCT
MC FADDEN(S-R)P142-152    BAUM P49B-68A    HOWES P61-67 (F-R)
SELBY P146-152
   28   29   30   31
DEFINITION OF RELATION
MC FADDEN(S-R)P165-184    HOWES P67-76(F-R)    SELBY P153-156.
176-207
   32   33   34   35
REFLEXIVE, SYMETRIC, TRANSITIVE PROPERTIES
SELBY P156-160    BAUM P68A-74B
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SETS, RELATIONS, AND FUNCTIONS INPUT CONTINUED

36 37 38 39
DEFINITION OF FUNCTION
MC FADDEN(S-R) P187-197 HOWES P76-103(F-R) BAUM P521A-515B
SELBY P209-224

40 41 42 43 44
RANGE AND DOMAIN
MC FADDEN (S-R)P197-217 BAUM 515B-527A HOWES P104-124(F-R)

45 46 47
FUNCTIONAL VALUES
MC FADDEN(S-R)P217-219 BAUM527A-540A HOWES(F-R)P82-86

48
SKETCH OF IDENTITY FUNCTION
MC FADDEN (S-R) P235-242

49
SKETCH OF THE CONSTANT FUNCTION
HOWES(F-R) P219-224

50
SKETCH OF ABSOLUTE VALUE
MC FADDEN (S-R) P235-242

51 52 53
INVERSE FUNCTION
MC FADDEN(S-R) P243-255 HOWES(F-R) &78-92,460-464

54 55 56 57
COMPOSITE FUNCTION
MC FADDEN (S-R) P219-230 HOWES P124-146(F-R) SELBY 251-256

58 59 60
SUMS, PRODUCTS, QUOTIENTS OF FUNCTIONS
HOWES P124-134 SELBY P234-247

*SETS RELATIONS FUNCTION PRETEST & RESOURCE MATERIALS**NOV 1970**
**********SETS UNIT BIBLIOGRAPHY**********
MCFADDEN, MOORE, AND SMITH - SETS, RELATIONS, AND FUNCTIONS
HOWES, VERNON E. - PRE-CALCULUS MATHEMATICS,FUNCTIONS, AND RELATIONS
BAUM AND DOBYNS - THE STRUCTURE OF THE REAL NUMBER SYSTEM
SELBY AND SWEET - SETS, RELATIONS AND FUNCTIONS

OBJECTIVES FOR ELEMENTARY FUNCTIONS

I Trigonometric Functions

1. The student should know the underlying properties of right triangles including the Pythagoren Theorem. (Pretest question 1,2)

2. The student should know the relationship of central angle, arc and radius. (Pretest questions 3, 4, 5)

3. The student should know the definitions of the six trigonometric functions. (Pretest questions 6, 7, 8, 9, 10, 11)

4. The student should know the values of the functions for the special angles; 30°, 45°, 60°, 90°, 180°, 270°. (Pretest questions 12, 13, 14, 15)

5. The student should be able to express the measure of an angle in either degrees or radians. (Pretest questions 16, 17, 18)

6. The student should be able to recogonize the sketch of a trigonometric function. (Pretest questions 19, 20, 21, 22)

7. The student should be able to indicate the period, range, and domain of a trigonometric function. (Pretest questions 23,24, 25, 26)

8. The student should be able to recogonize and use the basic trigonometric identities. (Pretest questions 27, 28, 29, 30, 31, 32)

9. The student should be able to recogonize the sketch of an inverse trigonometric function and be able to evaluate expressions involving inverse trigonometric functions. (Pretest questions 33, 34, 35,36)

10. The student should be able to determine values of the trigonometric functions for angles not in the first quadrant. (Pretest questions 37 - 40)

II EXPONENTIAL AND LOGRITHMIC FUNCTIONS

11. The student should know the meaning of negative, zero and fractonal exponents. (Pretest questions 41, 42, 43, 44)

12. The student should be able to apply the various laws of exponents. (Pretest questions 45, 46, 47, 48)

13. The student should know the sketch, range, and domain of the exponential function. (Pretest questions 54, 55, 56)

14. The student should know the properties of the log function and be able to evaluate the log of a number for a given base. (Pretest questions 49, 50, 51, 52, 53)

15. The student should know the sketch, range and domain of the log function. (Pretest questions 57, 58, 59)

16. The student should know that the log and exponential functions are inverse functions. (Pretest question 60)

Elementary Functions Pretest

1. A right triangle has legs of lengths 5 and 12. The hypotenuse has length
   (1) 13
   (2) 169
   (3) 17
   (4) none of these
   (5) I don't know

2. The hypotenuse of a right triangle has length x and one of the legs has length y. The othe leg therefore has length
   (1) $\sqrt{x^2 + y^2}$
   (2) $\sqrt{y^2 - x^2}$
   (3) $\sqrt{x^2 - y^2}$
   (4) none of these
   (5) I don't know

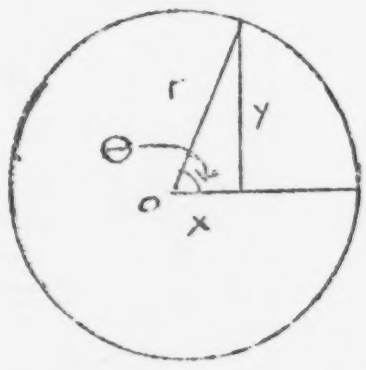
3. The percentage of the circumference of a circle cut out by an interior angle of 60° is
   (1) 1/30
   (2) 1/9
   (3) 1/6
   (4) 1/3
   (5) I don't know

4. The ratio of the circumference to the radius of a circle is
   (1) 2
   (2) 1
   (3) π
   (4) 2π
   (5) I don't know

5. In a circle with radius 1, an interior angle of 45° cuts an arc of length
   (1) $\pi/3$
   (2) $\pi/4$
   (3) $\pi/6$
   (4) $\pi/3$
   (5) I don't know

For questions 6 through 11, refer to the diagram at the right where you have a right triangle with an acute angle of θ degrees and sides with lengths as shown.

6. $\sin \theta =$
   (1) x/r
   (2) y/x
   (3) y/r
   (4) r/x
   (5) I don't know



7. $\tan \theta$ =
   (1) y/x
   (2) y/r
   (3) x/r
   (4) r/x
   (5) I don't know

8. $\cos \theta$ =
   (1) x/r
   (2) y/r
   (3) y/x
   (4) r/x
   (5) I don't know

9. $\sec \theta$ =
   (1) $1/(\sin \theta)$
   (2) $\sin (1/\theta)$
   (3) $1/(\cos \theta)$
   (4) $\cos (1/\theta)$
   (5) I don't know

10. $\csc \theta$ =
   (1) r/y
   (2) y/x
   (3) x/r
   (4) y/r
   (5) I don't know

11. $\cot \theta$ =
   (1) $(\sin \theta)/(\cos \theta)$
   (2) $\tan (1/\theta)$
   (3) $(\sec \theta) / (\csc \theta)$
   (4) $(\cos \theta)/(\sin \theta)$
   (5) I don't know

12. $\tan 45^\circ$ =
   (1) 0
   (2) 1/2
   (3) $\sqrt{2}/2$
   (4) 1
   (5) I don't know

13. $\sin 90^\circ$ =
   (1) 0
   (2) 1/2
   (3) $\sqrt{2}/2$
   (4) 1
   (5) I don't know

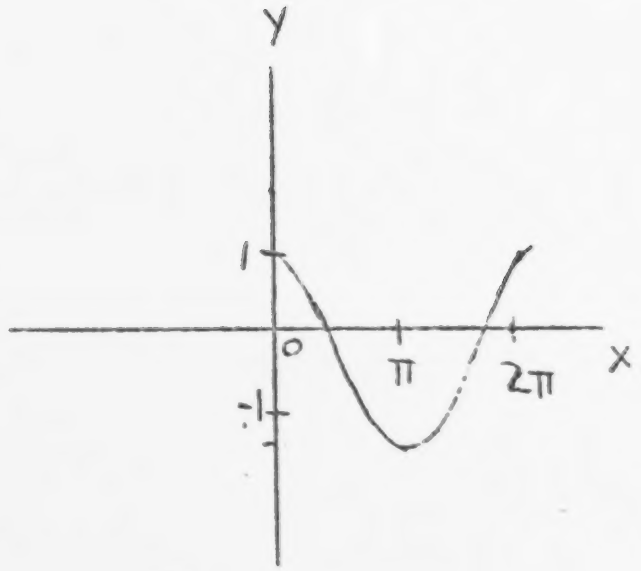
14. $\sin 30^\circ$ =
   (1) 1/2
   (2) $\sqrt{2}/2$
   (3) $\sqrt{3}/2$
   (4) 1
   (5) I don't know

15. The tangent function is zero at (in radians)
(1) 0 and π
(2) 0 and $\pi/2$
(3) $\pi/2$ and π
(4) $\pi/2$ and $-\pi/2$
(5) I don't know

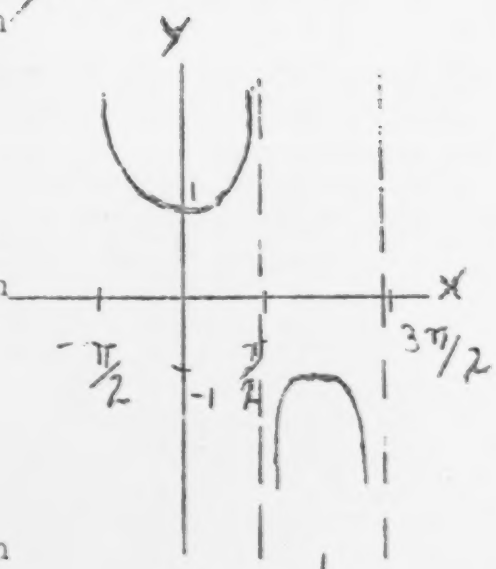
16. 180° in radian measure is
(1) 2
(2) 2π
(3) π
(4) $\pi/2$
(5) I don't know

17. 45° in radian measure is
(1) $45/\pi$
(2) $\pi/2$
(3) $\pi/4$
(4) $3\pi/4$
(5) I don't know

18. 2 radians is the same as
(1) 200°
(2) 30°
(3) 60°
(4) $(360/\pi)^\circ$
(5) I don't know

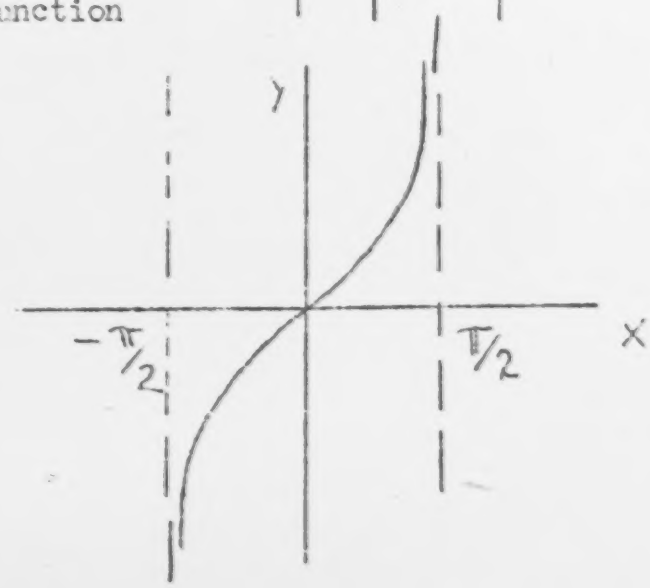


19. The graph at the right represents the function
(1) y = sin x
(2) y = cos x
(3) y = 2 sin x
(4) y = cos 2x
(5) I don't know

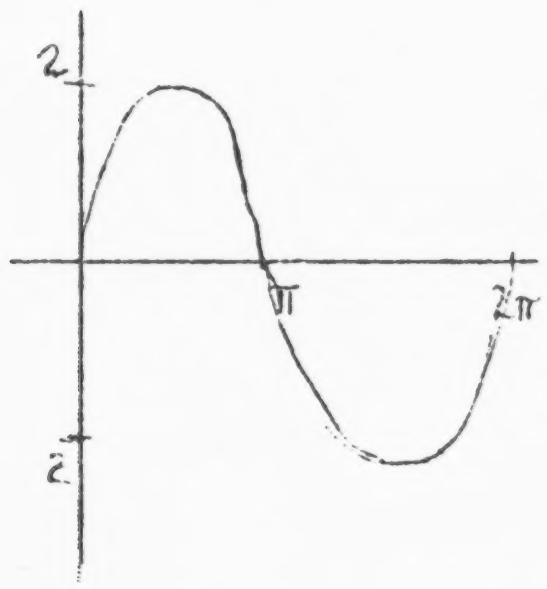


20. The graph at the right represents the function
(1) y = sin x
(2) y = cos x
(3) y = sec x
(4) y = csc x
(5) I don't know

21. The graph at the right represents the function
(1) y = cos x
(2) y = cotangent x
(3) y = tan x
(4) y = sec x
(5) I don't know



22. The graph at the right represents the function
(1) $y = \cos 2x$
(2) $y = 2 \cos x$
(3) $y = \sin 2x$
(4) $y = 2\sin x$
(5) I don't know



23. The period of the function $y = \cos x$ is
(1) 1
(2) 2
(3) π
(4) 2π
(5) I don't know

24. The period of the function $y = \tan x$ is
(1) 1
(2) 2
(3) π
(4) 2π
(5) I don't know

25. The domain of the function $y = \sin x$ is
(1) $-1 \leq x \leq 1$
(2) $-\infty < x < \infty$
(3) $0 \leq x \leq 2\pi$
(4) $0 \leq x \leq \pi$
(5) I don't know

26. The range of the function $y = \tan x$ is
(1) $0 \leq y \leq \pi$
(2) $-\infty < y < \infty$
(3) $-\pi \leq y \leq \pi$
(4) $0 \leq y \leq 2\pi$
(5) I don't know

27. Tanx cos x =
(1) $\sec^2 x$
(2) cot x
(3) csc x tan x
(4) sin x
(5) I don't know

28. $\sin^2 x + \underline{R} = 1$ is an identity if R =
(1) $\csc^2 x$
(2) $\cos^2 x$
(3) cos x
(4) $\sin^2 x$
(5) I don't know

29. $\sec^2 x =$
(1) $1 + \tan^2 x$
(2) $1 - \csc^2 x$
(3) $1 + \cot^2 x$
(4) $1 - \cot^2 x$
(5) I don't know

30. $\sin (x + y) =$
(1) $\sin x \sin y + \cos x \cos y$
(2) $\sin x \cos y + \sin y \cos x$
(3) $-\sin x \sin y + \cos x \cos y$
(4) $\sin x \cos y - \sin y \cos x$
(5) I don't know

31. $\cos 2x =$
(1) $\cos^2 x + \sin^2 x$
(2) $\cos^2 x - \sin^2 x$
(3) $\cos^2 2x$
(4) $\sin x$
(5) I don't know

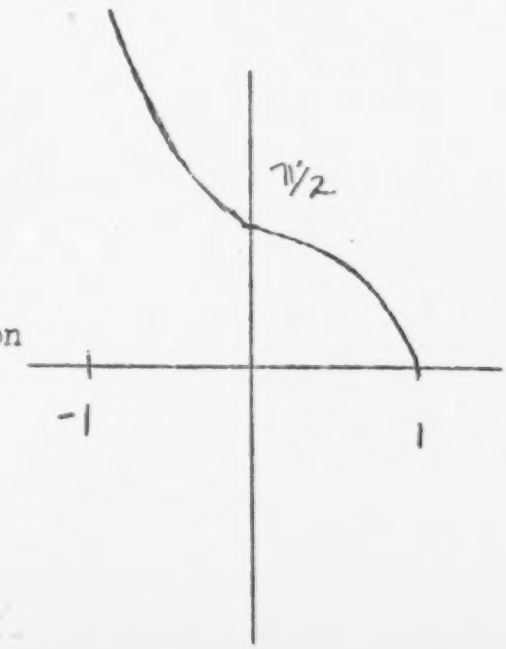
32. $\sin 40^\circ =$
(1) $2 \sin 20^\circ$
(2) $2 \cos 20^\circ$
(3) $4 \sin 20^\circ$
(4) $2 \sin 20^\circ \cos 20^\circ$
(5) I don't know

33. arc sin 1/2 (arc sin is the same as $\sin^{-1}$)
(1) 0°
(2) 30°
(3) 45°
(4) 60°
(5) I don't know

34. arc sin (sin $\pi/4$) =
(1) 1
(2) $\sqrt{2}/2$
(3) $\pi/4$
(4) $\pi/2$
(5) I don't know

35. If sin x = 1/3, then cos x =
(1) 2/3
(2) 1/6
(3) $2\sqrt{2}/3$
(4) 8
(5) I don't know

36. The graph at the right represents the function
(1) y = cot x
(2) y arc sin x
(3) y = arc cos x
(4) y = arc tan x
(5) I don't know



37. Indicate the correct answer
(1) sin 125° is positive
(2) cos 125° is positive
(3) tan 125° is positive
(4) none of the above
(5) I don't know

38. Indicate the correct answer
(1) sin 190° = sin 10°
(2) sin 190° = cos 10°
(3) sin 190° = - sin 10°
(4) sin 190° = -cos 10°
(5) I don't know

39. cos 30° is the same as
(1) sin 60°
(2) cos 270°
(3) cos 60°
(4) cos (-60°)
(5) I don't know

40. sin (-30°) is the same as
(1) sin 30°
(2) cos 30°
(3) -cos 30°
(4) -sin 30°
(5) I don't know

In questions 41, 42, and 43, x,y, and z are positive real numbers.

41. x^{y+z} =
(1) $x^y + x^z$
(2) $x^y x^z$
(3) x^{yz}
(4) $(x^y)^z$
(5) I don't know

42. x^{yz} =
(1) $x^y + x^z$
(2) $x^y x^z$
(3) x^{y+z}
(4) $(x^y)^z$
(5) I don't know

43. If $x \neq 0$ then $\frac{x^y}{x^z}$ equals
(1) x^{yz}
(2) x^{-yz}
(3) x^{y-z}
(4) $x^{y/z}$
(5) I don't know

44. $\frac{(x^3)^2 x^4}{x^5}$ =

(1) x^8
(2) x^4
(3) x^5
(4) x^{15}
(5) I don't know

45. 4^0 =

(1) 0
(2) 1
(3) 2
(4) 4
(5) I don't know

46. 3^{-2} =

(1) 1/3
(2) -9
(3) -6
(4) 1/9
(5) I don't know

47. $8^{-2/3}$ =

(1) 1/4
(2) 4
(3) 16/3
(4) -16/3
(5) I don't know

48. $(64)^{3/2}$ =

(1) 16
(2) 32
(3) 512
(4) 1024
(5) I don't know

49. Log_{10} 100 =

(1) 1
(2) 2
(3) 3
(4) 4
(5) I don't know

50. Log_2 8 =

(1) 16
(2) π
(3) 4
(4) 3
(5) I don't know

In questions 51 and 52, x,y, and z are positive numbers.

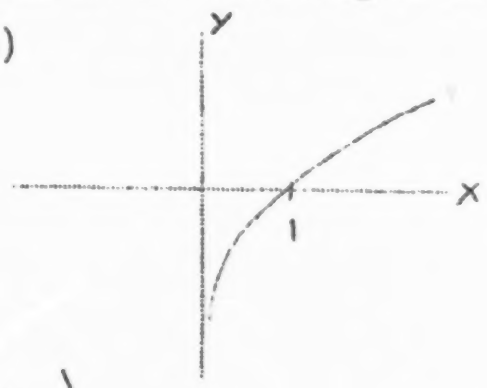
51. Log xy =
    (1)(log x)(log y)
    (2) x(logy)
    (3) log x + log y
    (4) $(\log x)^y$
    (5) I don't know

52. Log x^y =
    (1) (log x)(log y)
    (2) y log x
    (3) x log y
    (4) $(\log x)^y$
    (5) I don't know

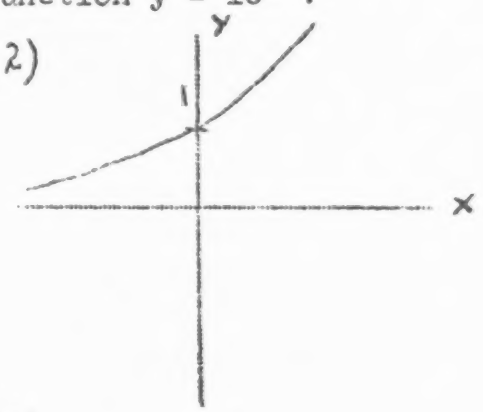
53. Log 8 =
    (1) 3 log 2
    (2) $(\log 4)^2$
    (3) $(\log 2)^3$
    (4) (log 2)(log 4)
    (5) I don't know

54. Which of the following is the graph of the function $y = 10^x$?

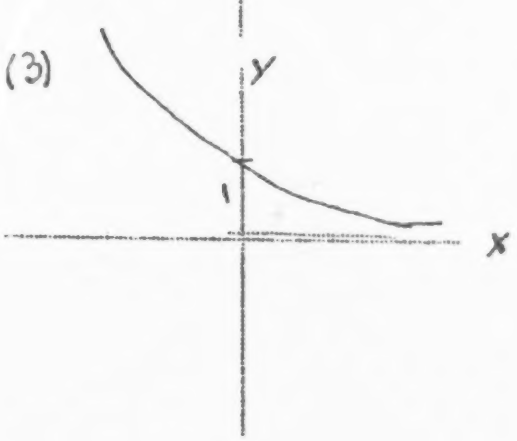
(1)



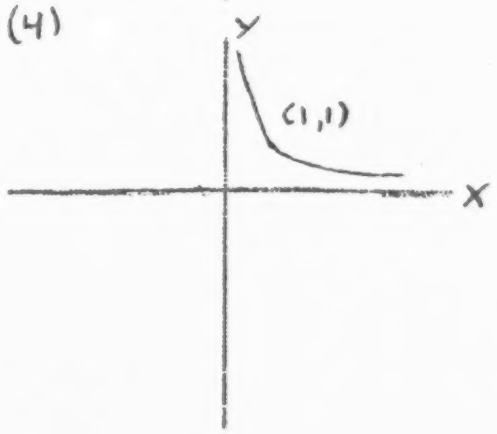
(2)



(3)



(4)



(5) I don't know

55. The range of the function $y = 10^x$ is
    (1) $0 \leq r < \infty$
    (2) $-\infty < r < \infty$
    (3) $0 \leq r \leq 1$
    (4) $-1 \leq r \leq 1$
    (5) I don't know

56. The domain of the function $y = 10^x$ is
(1) $0 \leq d < \infty$
(2) $-\infty < d < \infty$
(3) $0 \leq d \leq 1$
(4) $-1 \leq d \leq 1$
(5) i don't know

57. The domain of the function $y = \log x$ is
(1) $0 \leq d < \infty$
(2) $-\infty < d < \infty$
(3) $0 \leq d \leq 1$
(4) $-1 \leq d \leq 1$
(5) I don't know

58. The range of the function $y = \log x$ is
(1) $0 \leq d < \infty$
(2) $-\infty < d < \infty$
(3) $0 \leq d \leq 1$
(4) $-1 \leq d \leq 1$
(5) I don't know

59. Which of the graphs presented in #54 represents the function $y = \log x$?
(1) 1
(2) 2
(3) 3
(4) 4
(5) I don't know

60. The inverse of the function $y = 10^x$ is the function
(1) $y = x^{10}$
(2) $y = x^{1/10}$
(3) $y = \log_{10} x$
(4) $y = \log 10^x$
(5) I don't know

ANS. KEY 133423113144411334233443224212242333131424332413243212121213
115443640 133451115144451335233443535255555515131555552411153252155555

30 RIGHT 5 WRONG 25 DONT KNOW

PYTHAGORGAN THEOREM AND CIRCLE RELATIONSHIPS
4 RIGHT 0 WRONG 1 DONT KNOW OUT OF 5

DEFINATION OF TRIG FUNCTIONS
4 RIGHT 1 WRONG 1 DONT KNOW OUT OF 6

SPECIAL ANGLES-30,45,60,90,180,270
3 RIGHT 0 WRONG 1 DONT KNOW OUT OF 4

RADIAN MEASURE
2 RIGHT 0 WRONG 1 DONT KNOW OUT OF 3

GRAPHS OF TRIG FUNCTIONS
4 RIGHT 0 WRONG 0 DONT KNOW OUT OF 4

PERIOD,RANGE,DOMAIN OF TRIG FUNCTIONS
2 RIGHT 1 WRONG 1 DONT KNOW OUT OF 4

IDENTITIES
1 RIGHT 0 WRONG 5 DONT KNOW OUT OF 6
MC FADDEN (TRIG) P189-220,226,232,239-250 HOWES(TRIG) P238-320

INVERSE TRIG FUNCTIONS
0 RIGHT 1 WRONG 3 DONT KNOW OUT OF 4
MC FADDEN (TRIG) P597-636 HOWES (TRIG) P361-412

ANGLES NOT IN FIRST QUADRANT
3 RIGHT 0 WRONG 1 DONT KNOW OUT OF 4

BASIC LAWS OF EXPONENTS
0 RIGHT 0 WRONG 4 DONT KNOW OUT OF 4
REIGH P327-349 REES P57-59 SWOKOWSKI P30-41

ZERO,NEGATIVE AND RATIONAL EXPONENTS
3 RIGHT 1 WRONG 0 DONT KNOW OUT OF 4

PROPERTIES OF LOG FUNCTIONS
2 RIGHT 1 WRONG 2 DONT KNOW OUT OF 5

SKETCH,RANGE,DOMAIN OF EXPONENTIAL FUNCTIONS
2 RIGHT 0 WRONG 1 DONT KNOW OUT OF 3

SKETCH,RANGE,DOMAIN OF LOG FUNCTION
0 RIGHT 0 WRONG 3 DONT KNOW OUT OF 3
SWOKOWSKI P138-141 DOBYNS 135 A-137 C REES P251-253

INVERSE OF THE EXPONENTIAL FUNCTION
0 RIGHT 0 WRONG 1 DONT KNOW OUT OF 1
INVERSE OF EXPONENTIAL FUNCTION INCORRECT

ELEMENTARY FUNCTICNS UNIT BIBLIOGRAPHY

REES, PAUL AND FRED SPORKS ALGEBRA,TRIGONOMETRY AND ANALYTIC GEOMETRY

REIGH, MILDRED AND WILIAM HAUCK ALG. REVIEW MANUAL

HOWES, VERNON E. ANALYTIC TRIGONOMETRY

MC FADDEN, MYRA MODERN TRIGONOMETRY- A PROGRAM FOR SELF INSTRUCTION

SWOKOWSKI, CARL FUNDAMENTALS OF ALGEBRA AND TRIGONOMETRY

DOBYNS, ROY A PROGRAMED SUPPELEMENT TO FUNDAMENTALS OF ALGEBRA AND TRI-

ELEMENTARY FUNCTIONS INPUT FOR INFOS

```
// JOB     CCC1
// XEQ INFCS
  60  15   7
  5   6   4   3   4   4   6   4   4   4   4   5   3   3   1
  3   2   2   2   2   2   3   3   2   2   2   2   2   2   1
    1     2     3     4     5
PYTHAGOREAN THEOREM AND CIRCLE RELATIONSHIPS
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MC FADDEN (TRIG) P19-23 —— HOWES (TRIG) P41-50

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    6     7     8     9     10    11
DEFINITION OF TRIG FUNCTIONS
MC FADDEN (TRIG) P32-58 , P221-226 —— HOWES (TRIG) P85-109,
P192-202
    12    13    14    15
SPECIAL ANGLES-30,45,60,90,180,270
MC FADDEN (TRIG) P66-75 —— HOWES (TRIG) P436-454

    16    17    18
RADIAN MEASURE
MC FADDEN (TRIG) P22-32 —— HOWES P413-435

    19    20    21    22
GRAPHS OF TRIG FUNCTIONS
MC FADDEN(TRIG) P122-138 —— HOWES P137-190,214-236 SWO
KOSKI P173-178 —— COBYNS 178 B-181 A FAWCETT P168-179
    23    24    25    26
PERIOD,RANGE,DOMAIN OF TRIG FUNCTIONS
MC FADDEN (TRIG)P341-408

    27    28    29    30    31    32
IDENTITIES
MC FADDEN (TRIG) P189-220,226,232,239-250 —— HOWES(TRIG) P238-320

    33    34    35    36
INVERSE TRIG FUNCTIONS
MC FADDEN (TRIG) P597-636 —— HOWES (TRIG) P361-412
```

ELEMENTARY FUNCTIONS INPUT CONTINUED

37 38 39 40
ANGLES NOT IN FIRST QUADRANT
MC FADDEN(TRIG) P85-116 HOWES (TRIG) P110-136

41 42 43 44
BASIC LAWS OF EXPONENTS
REIGH P327-349 REES P57-59 SWOKOWSKI P30-41

45 46 47 48
ZERO,NEGATIVE AND RATIONAL EXPONENTS
REIGH P349-373 REES P59-67

49 50 51 52 53
PROPERTIES OF LOG FUNCTIONS
REIGH P555-573 REES P241-250 SWOKOWSKI P133-137 DOBYNS P 130A-134D

54 55 56
SKETCH,RANGE,DOMAIN OF EXPONENTIAL FUNCTIONS
SWOKOSKI P129-133 DOBYNS 124 A-129 C REES P251-253

57 58 59
SKETCH,RANGE,DOMAIN OF LOG FUNCTION
SWOKOWSKI P138-141 DOBYNS 135 A-137 C REES P251-253

60
INVERSE OF THE EXPONENTIAL FUNCTION

INVERSE OF EXPONENTIAL FUNCTION INCORRECT

***ELEMENTARY FUNCTIONS PRETEST AND RESOURCE MATERIALS*
ELEMENTARY FUNCTIONS UNIT BIBLIOGRAPHY
REES, PAUL AND FRED SPARKS ALGEBRA,TRIGONOMETRY AND ANALYTIC GEOMETRY
REIGH, MILDRED AND WILLIAM HAUCK ALG. REVIEW MANUAL
HOWES, VERNON E. ANALYTIC TRIGONOMETRY
MC FADDEN, MYRA MODERN TRIGONOMETRY- A PROGRAM FOR SELF INSTRUCTION
SWOKOWSKI, CARL FUNDAMENTALS OF ALGEBRA AND TRIGONOMETRY
DOBYNS, ROY A PROGRAMED Supplement TO FUNDAMENTALS OF ALGEBRA and Trig.

Objectives for Algebra Unit

1. The student should be able to add, subtract, multiply and divide polynomials. (Pretest questions 1, 2, 3, 4, 5, 6)

2. The student should be able to perform basic operations on algebraic fractions. (Pretest questions 7, 8, 9, 10)

3. The student should be able to simplify algebraic expressions involving rational exponents and radicals. (Pretest questions 11, 12, 13, 14, 15, 16, 17)

4. The student should be able to apply the remainder theorem and factor theorem. (Pretest questions 18, 19, 20)

5. The student should be able to apply the distributive property to find common monomial or binomial factors. (Pretest questions 21, 22, 23, 24, 25)

6. The student should be able to solve quadratic equations by factoring, and by using the quadratic formula and be able to use the discriminant to determine the nature of the roots. (Pretest questions 26, 27, 28, 29)

7. The student should be able to solve equations containing absolute values and simple inequalities involving absolute value. (Pretest questions 30, 31, 32, 33)

8. The student should know how to rationalize the denominator or numerator of a fraction involving irrational numbers. (Pretest questions 34, 35, 36, 37)

9. The student should be able to solve quadratic equations by completing the square. (Pretest questions 38, 39, 40)

10. The student should be able to solve irrational and fractional equations. (Pretest questions 41, 42, 43)

11. The student should know the relationships existing between roots and coefficients of polynominal equations. (Pretest questions 44, 45, 46)

12. The student should be able to solve systems of equations. (Pretest questions 47, 48, 49)

**********ALGEBRA UNIT BIBLIOGRAPHY**********

ALWIN AND HACKWORTH - ALGEBRA REVIEW

DAVIS, THOMAS - ANALYTIC GEOMETRY

DOBYNS, ROY - A PROGRAMED SUPPLEMENT TO FUND. OF ALGEBRA AND TRIG BY SWOKOWSKI

HOWES, VERNON E. - PRE-CALCULUS MATHEMATICS - ALGEBRA

REIGH AND HAUCK - BRIEF ALGEBRA REVIEW

SWOKOSKI, EARL - FUNDAMENTALS OF ALGEBRA AND TRIGONOMETRY

Álgebra Pretest

In problems 1-14 perform the indicated operations and simplify.

1. $(3x^4-2x) + (x^5-x^4)$

   1) x^5+2x^4-2x

   2) $4x^9-3x^5$

   3) x^3+x

   4) $2x^4$

   5) I do not know

2. $(3x^2)(x^4-2x^3)$

   1) $-3x^8+6x^6$

   2) $-3x^6+6x^5$

   3) $-3x^6-6x^5$

   4) $-3x^6-2x^3$

   5) I do not know

3. $\frac{3x^4y^2-2x^2y^5+x^2y^2}{x^2y^2}$

   1) $3x^4y^2-2x^2y^5$

   2) $3x^2-2y^3$

   3) $3x^2-2xy^3+1$

   4) $3x^2-2y^3+1$

   5) I do not know

4. $(x+2)(x-3)$

   1) x^2-6

   2) x^2-x-5

   3) x^2-x-6

   4) x^2-5x-6

   5) I do not know

y^3

5. $(x+y)^3 =$

1) $x^3+3xy+y^3$

2) $x^3 + y^3$

3) $x^3 + 3x^2y + 3xy^2 + y^3$

4) $x3 + x^2y + xy^2 + y^3$

5) I do not know

6. $\frac{2x(x + 4) - (2x^2 + 7x - 2)}{(x + 2)^2}$

1) $\frac{1}{x + 2}$

2) $\frac{4x^2 + 15x - 2}{(x + 2)^2}$

3) $\frac{15x - 2}{(x + 2)^2}$

4) $\frac{1}{(x + 2)^2}$

5) I do not know

7. $\frac{(x + 2)^3}{(x - 3)} \cdot \frac{(x + 2)}{(x - 3)^4}$

1) $\frac{(x + 2)^4}{(x - 3)^5}$

2) $(x + 2)^2 \ (x - 3)^3$

3) $\frac{1}{(x + 2)^2 \ (x - 3)^3}$

4) None of the first three choices

5) I do not know

8. $\frac{1}{x+2} + \frac{x}{x+6}$

1) $\frac{x + 1}{2x + 8}$

2) $\frac{x + 8}{6(x + 2)}$

3) $\frac{2x + 8}{(x+2)(x+6)}$

4) $\frac{x^2 + 3x + 6}{(x + 2)(x + 6)}$

9. $\frac{1/x - 1/2}{x - 2}$

1) $\frac{1}{2X}$

2) $\frac{(2 - x)(x - 2)}{2x}$

3) $-1/2x$

4) 0

5) I do not know

10. $\frac{(y + 3)^2}{(y^2 - 1)} \times \frac{(y - 1)^2}{(y + 3)} =$

1) $y + 3$

2) $\frac{(y + 3)^3}{(y^2 - 1)(y - 1)^2}$

3) $\frac{(y + 3)(y - 1)}{y + 1}$

4) none of the first three choices

5) I do not know

11. $(36x^{16})^{1/2} =$

1) $18x^8$

2) $18x^4$

3) $6x^8$

4) $6x^4$

5) I do not know

12. $\sqrt{x+1} \ (\sqrt[3]{x} + \sqrt{x + 1})$

1) $\sqrt[6]{x(x + 1)} + \sqrt{x + 1}$

2) $\sqrt[3]{x(x + 1)} + x + 1$

3) $\sqrt[2]{x(x + 1)^3} + \sqrt{x + 1}$

4) $\sqrt{x + 1} \ \sqrt[3]{x} + x + 1$

5) I do not know

13. $x^{3/2}(x^{1/2} + x^{5/2})$

1) $x^2 + x^4$

2) x^6

3) $x^{3/4} + x^{15/4}$

4) $x^{18/4}$

5) I do not know

14. $4x^{-4} + 2x^{-2}$

1) $6x^{-6}$

2) $\dfrac{2(2 + x^2)}{x^{-4}}$

3) $\dfrac{2(2 + x^2)}{x^4}$

4) $\dfrac{2(2 + x^2)}{x^2}$

5) I do not know

15. Write $x^{-4/3}$ in an equivalent radical form .

1) $\sqrt[3]{x^4}$

2) $-\sqrt[3]{x^4}$

3) $\dfrac{1}{\sqrt[3]{x^4}}$

4) $\dfrac{1}{\sqrt[4]{x^3}}$

5) I do not know

16. Write $\sqrt[7]{(x - 5)^5}$ in an equivalent form using fractional exponents.

1) $(x - 5)^{7/5}$

2) $(x - 5)^{5/7}$

3) $(x - 5)^2$

4) $(x - 5)^{-2}$

5) I do not know

17. Write $(x^2 + 1)^{1/2}(2x) + x(x^2 + 1)^{-1/2}$ in an equivalent form in which there are no negative exponents.

1) $(x^2 + 1)^{1/2}(2x) + x(x^2 + 1)^{1/2}$

2) $x(2x^2 + 3)$

3) $3x(x^2 + 1)^{1/2}$

4) $\dfrac{x(2x^2 + 3)}{(x^2 + 1)^{1/2}}$

5) I do not know

18. Since 2 is a root of the equation $x^3 + x^2 - 5x - 2 = 0$, a factor of $x^3 + x^2 - 5x - 2$ is:

1) $2x - 1$

2) $x + 2$

3) $x - 2$

4) $x + 1$

5) I do not know

19. When $x^3 - 2x^2 + 3x + 1$ is divided by $x - 2$ the remainder is:

1) 0
2) 7
3) 1
4) $x - 2$
5) I do not know

20. The roots of the equation $(x + 2)(x - 1)^2 = 0$ are:

1) 1 -1 -2
2) 1, 1, 2
3) 1, 1, -2
4) -1, -1, 2
5) I do not know

21. Factor completely $3x^2y + 6xy + 3xy^2$

1) $3x(xy + 2y + y^2)$
2) $3y(x^2 + 2x + x)$
3) $xy(3x + 6 + 3y)$
4) $3xy(x + 2 + y)$
5) I do not know

22. Factor $x^2 - 9$

1) $x - 3$
2) $(x - 3)(x - 3)$
3) $(x + 3)(x - 3)$
4) $(x + 9)(x - 9)$
5) I do not know

23. Factor $x^2 + x - 6$

1) $(x + 2)(x - 3)$
2) $(x - 6)(x + 1)$
3) $(x + 6)(x - 1)$
4) $(x - 2)(x + 3)$
5) I do not know

24. Simplify $2(x + 2)^{1/2} (x-5) + (x+2)^{1/2} (x+1)$

1) $(x + 2)^{1/2} (3x - 9)$

2) $2(x + 2)^{1/2} (2x - 4)$

3) $2(x + 2)^{1/2} (x - 5) (x + 1)$

4) None of the first three choices

5) I do not know

25. Simplify $\dfrac{x^2 - 6x + 5}{x - 1}$

1) $x + 5$
2) $x - 6$
3) $x - 1$
4) $x - 5$
5) I do not know

26. The solution set for the equation $x(x - 4) = 0$ is:

1) $x = 0 \quad x = 4$
2) $x = 0 \quad x = -4$
3) $x = 2, \; x = -2$
4) $x = 4, \; x = 1$
5) I do not know

27. The solutions of the quadratic equation $ax^2 + bx + c \quad 0$ are:

1) $x = \dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

2) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3) $x = \dfrac{b \pm \sqrt{b^2 + 4ac}}{a}$

4) $x = \dfrac{-b \pm \sqrt{b^2 + 4ac}}{a}$

5) I do not know

28. If $x = 2 + i$ is a solution of the equation $ax^2 + bx+c=0$ where a,b,c are integers then the other solution is:

1) $x = 2 + i$

2) $\dfrac{1}{2 + i}$

3) $2 - i$

4) It is not possible to determine the other solution.

5) I do not know.

29. The quadratic equation $ax^2 + bx + c = 0$ will have two equal roots if:

1) $b^2 - 4ac = 25$

2) $b^2 - 4ac = 23$

3) $b^2 - 4ac = 0$

4) $b^2 - 4ac = -25$

5) I do not know

30. If $|x + 4| = 7$ then x equals:

1) 3, 10
2) 11 -3
3) 3 -11
4) 3 only
5) I do not know

31. If $|x| < 5$ then:

1) $-5 < x < 5$
2) $0 < x < 5$
3) $x > 5$ or $x < 5$
4) $-5 \leq x \leq 5$
5) I do not know

32. The values of x for which $-2x + 6 > 2$ are:

1) $x < -2$
2) $-2 < x < 2$
3) $x > 2$
4) $x < 2$
5) I do not know

33. The values of x for which $|x - 2| < 1$ are:

1) $x < 1$ or $x > 3$
2) $1 \leq x \leq 3$
3) $1 < x < 3$
4) $x > 3$
5) I do not know

34. Rationalizing the denominator of the fraction $\frac{3}{\sqrt{2} - 1}$ gives:

1) $\frac{\sqrt{2} - 1}{3}$

2) $\frac{\sqrt{2} + 1}{3}$

3) $3(\sqrt{2} + 1)$

4) $3(\sqrt{2} - 1)$

5) I do not know

35. Rationalizing the numerator of the fraction $\frac{\sqrt{x + 1}}{5}$ gives:

1) $\sqrt{\frac{x - 1}{5}}$

2) $\frac{5}{\sqrt{x} + 1}$

3) $\frac{x - 1}{5(\sqrt{x} - 1)}$

4) none of the first three choices

5) I do not know

36. To rationalize the denominator of the fraction $\frac{\sqrt{x}}{\sqrt{x + a} - x}$, multiply both numerator and denominator by:

1) $\sqrt{x}$

2) $\sqrt{x + a}$

3) $\sqrt{x + a} + x$

4) $\sqrt{x + a} - x$

5) I do not know

37. Write $\frac{2 + \sqrt{x}}{x}$ in an equivalent form without a radical in the numerator.

1) $\frac{4 - x}{x}$

2) $\frac{x}{2 + \sqrt{x}}$

3) $\frac{x}{x(2 + \sqrt{x})} \times \frac{4 + x}{(2 + \sqrt{x})}$

4) $\frac{4 - x}{x(2 - \sqrt{x})}$

5) I do not know

38. If the trinomial $x^2 - 6x + k$ is a perfect square then k

1) 3
2) 9
3) 36
4) -3
5) I do not know

39. Writing $x^2 + 2x + 2$ in the form $(x + b)^2 + a$ by completing the square yields:

1) $(x + 1)^2 + 1$
2) $(x + 1)^2 - 1$
3) $(x + 2)^2 + 2$
4) $(x - 2)^2 + 1$
5) I do not know

40. If $(x - b)^2 = a$ then:

1) $x = b \pm \sqrt{a}$
2) $x = -b \pm \sqrt{a}$
3) $x = a \pm \sqrt{b}$
4) $x = -a \pm \sqrt{b}$
5) I do not know

41. The solution set for the equation $\sqrt{x + 2} = x$ is:

1) {2}
2) {-1, -2}
3) {1, -2}
4) {-1, 2}
5) I do not know

42. To solve the equation $\frac{-5}{x + 2} + 3 = \frac{4}{x + 2}$ first

1) add x + 2 to both sides of the equation
2) divide both sides of the equation by x + 2
3) multiply both sides of the equation by x + 2
4) divide both sides of the equation by 3

43. The equation $\frac{(x + 1)(x - 2)}{\sqrt{x^2 - 4}} = 0$ has

1) exactly one solution
2) exactly two solutions
3) no solutions
4) exactly three solutions'
5) I do not know

44. Solve $x y^2 z + yz^2 = xz$ for x in terms of y and z

1) $x = \frac{-yz}{y^2 - 1}$
2) $x = \frac{x - yz}{y^2}$
3) $x = -yz^2 - y^2z + z$
4) None of the first three choices
5) I do not know.

45. The product of the roots of the equation $ax^2 + bx + c = 0$ is:

1) b/a
2) a/b
3) a/c
4) c/a
5) I do not know

46. Which of the following statements may be made about the roots of the cubic equation $ax^3 + bx^2 + cx + d=0$

1) it has at least one real root
2) it has at least two real roots
3) it has three real roots
4) it has at most one real root
5) I do not know

47. The system of equations $x + 3y = -1$, $2x - y = 5$ has a solution (x,y) where:

1) x and y are positive
2) x is zero and y is positive
3) x is positive and y is negative
4) x is negative and y is positive
5) I do not know

48. If a, b c, are distinct non-zero constants which of the following pairs of equations has no solution?

1) $ax + by = c$
   $ax + by = c + 1$

2) $ax + by = c$
   $bx + ay = c$

3) $ax + by = c$
   $2ax + 2by = 2c$

4) $(a+1)x + by = c$
   $ax + by = c$

5) I do not know

49. The system of equations $x + 2y = 4$ / $x^2 + 4y^2 = 16$ has:

1) exactly one solution
2) exactly two solutions
3) no solutions
4) exactly four solutions
5) I do not know.

**********ALGEBRA PRE-TEST AND RESOURCE MATERIALS***

ANS. KEY 124331243334333243234341412333143333421113114131 2
051400745 122330255412152245254142422532134444531413255535 2

19 RIGHT 19 WRONG 11 DONT KNOW

SIMPLIFY ALGEBRAIC EXPRESSIONS
4 RIGHT 2 WRONG 0 DONT KNOW OUT OF 6

OPERATIONS ON ALGEBRAIC FUNCTIONS
1 RIGHT 1 WRONG 2 DONT KNOW OUT OF 4
REIGH P86-127 ALWIN P140-146,194-199,210-249 HOWES P229-260

RADICALS AND RATIONAL EXPONENTS
2 RIGHT 4 WRONG 1 DONT KNOW OUT OF 7
ALWINS P340-353,380-386 SWOKOSKI P27-42 DOBYNS 26A-398

REMAINDER AND FACTOR THEOREMS
1 RIGHT 0 WRONG 2 DONT KNOW OUT OF 3
HOWES P467-538 SWOKOWSKI P328-331 DOBYNS 3328-335D

DISTRIBUTIVE PROPERTY AND FACTORING
3 RIGHT 2 WRONG 0 DONT KNOW OUT OF 5

QUADRATIC EQUATIONS
2 RIGHT 1 WRONG 1 DONT KNOW OUT OF 4
ALWIN P297-314,441-460 HOWES(ALG)P277-287

ABSOLUTE VALUE AND INEQUALITY
1 RIGHT 3 WRONG 0 DONT KNOW OUT OF 4
T. DAVIS P2-48 REIGH P162-186,263,275 HOWES(ALG)P305-342

RATIONALIZE NUMERATOR AND DENOMINATOR
0 RIGHT 3 WRONG 1 DONT KNOW OUT OF 4
ALWIN P353-379

COMPLETING THE SQUARE
1 RIGHT 2 WRONG 0 DONT KNOW OUT OF 3
ALWIN P386-441 HOWES(ALG) P205-213

IRRATIONAL AND FRACTIONAL EQUATIONS
2 RIGHT 1 WRONG 0 DONT KNOW OUT OF 3

RELATIONSHIP BETWEEN ROOTS AND COEFFICIENTS
0 RIGHT 0 WRONG 3 DONT KNOW OUT OF 3
HOWES(ALG)P520-531

SYSTEMS OF EQUATIONS
[illegible] DONT KNOW OUT OF 3

ALGEBRA INPUT FOR INFOS

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// XEQ INFCS
  49  12   7
   6   4   7   3   5   4   4   4   3   3   3   3
   4   3   5   2   3   3   3   3   2   2   2   2
     1   2   3   4   5   6
SIMPLIFY ALGEBRAIC EXPRESSIONS
REIGH P1-46      ALWIN P115-131,146-168      SWCKCWSKI P42-47
DOBYNS 39C-44A
     7   8   9  1C
CPERATICNS CN ALGEBRAIC FUNCTICNS
REIGH P86-127      ALWIN P140-146,194-199,210-249      HOWES P229-260
I          DOBYNS 52A-56B  SWOKOSKI P52-55
    11  12  13  14  15  16  17
RADICALS AND RATICNAL EXPONENTS
ALWINS P34C-353,38C-386      SWCKCSKI P27-42      DOBYNS 26A-39B

    18  19  2C
REMAINDER AND FACTCR THECREMS
HOWES P467-538      SWCKCWSKI P328-331      DOBYNS 332B-335D

    21  22  23  24  25
DISTRIBUTIVE PROPERTY AND FACTCRING
REIGH P48-83      ALWIN P131-140,168-194,199-210      HOWES P177-228
          DOBYNS 44B-51C  SWOKOWSKI P48-51
    26  27  28  29
QUADRATIC EQUATICNS
ALWIN P297-314,441-46C      HOWES(ALG)P277-287

    30  31  32  33
ABSCLUTE VALUE AND INEQUALITY
T. DAVIS P2-48      REIGH P162-186,263,275      HOWES(ALG)P305-342
          DCBYNS 77B-89C  SWOKOSKI P75-83
    34  35  36  37
RATICNALIZE NUMERATCR AND DENCMINATCR
ALWIN P353-379

    38  39  4C
CCMPLETING THE SQUARE
ALWIN P386-441      HCWES(ALG) P205-213

    41  42  43
IRRATICNAL AND FRACTICNAL EQUATICNS
REIGH P187-196      ALWIN P272-296      HOWES(ALG) P290-304

    44  45  46
RELATICNSHIP BETWEEN RCOTS AND COEFFICIENTS
HOWES(ALG)P52C-531

    47  48  49
SYSTEMS CF EQLATICNS
REIGH P276-313      ALWIN P81-111

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69

Objectives for Analytic Geometry Unit

1. The student should recogonize the various forms of the equation of a straight line. (Pretest questions 1, 2, 3, 4, 5)

2. The student should be able to write the equation of a straight line, given sufficient information about the line. (Pretest questions 6, 7, 8, 9)

3. The student should be able to work with parallel and perpendicular lines. (Pretest questions 10, 11, 12, 13)

4. The student should be able to use the distance formula to determine the distance between two points and the distance from a point to a line. (Pretest questions 14, 15)

5. The student should be able to identify the conic represented by a second degree equation in two unknowns. (Pretest questions 16, 17, 18, 19, 20, 21, 22)

6. The student should be able to write the equation of a circle in standard form by completing the square and identify the center and radius of the circle. (Pretest questions 23, 24, 25, 26, 27, 28)

7. The student should know the locus definition of a parabola, recogonize the sketch of a parabola and be able to identify the vertex and focus of a parabola. (Pretest questions 29, 30, 31, 32, 33)

8. The student should know the locus definition of an ellipse, recogonize the sketch of an ellipse, and be able to identify the center, major axis and foci of an ellipse. (Pretest questions 34, 35, 36, 37)

9. The student should know the locus definition of a hyperbola, recogonize the sketch of a hyperbola, and be able to identify the center, vertices and asympotes of a hyperbola. (Pretest questions 38, 39, 40, 41)

10. The student should be able to indicate the asympotes and symmetries for a given curve. (Pretest questions 42, 43, 44, 45, 46)

Analytic Geometry Pretest

1. Which of the following is <u>not</u> the equation of a straight line where A,B,C are constants?
   (1) $AX + BY + C = 0$
   (2) $Y = AX + B$
   (3) $X/A + Y/B = 1$
   (4) $Y = A/X$
   (5) I do not know.

2. $Ax + By + C = 0$ is the equation of
   (1) a line whose slope is A/B
   (2) a line whose slope is $-A/B$
   (3) a line whose slope is B/A
   (4) a line whose slope is $-B/A$
   (5) I do not know

3. In the equation $y=mx + b$
   (1) b is the slope
   (2) b is the x-intercept
   (3) b is the y-intercept
   (4) This is not a straight line.
   (5) I do not know.

4. If $x/a + y/b = 1$ where $a \neq 0$, $b \neq 0$, then
   (1) a is the x-intercept
   (2) a is the y-intercept
   (3) a is neither the x-intercept nor the y-intercept
   (4) b is the x-intercept
   (5) I do not know.

5. Given $x/a + y/2a = 1$. This is
   (1) a line with the x-intercept equal to twice the y-intercept
   (2) a line with the y-intercept equal to twice the x-intercept
   (3) a family of lines with the x-intercept equal to twice the y-intercept
   (4) a family of lines with the y-intercept equal to twice the x-intercept
   (5) I do not know.

6. The equation of the line having slope 4 and y-intercept 3 is:
   (1) $y=4x + 3$
   (2) $y= 3x + 4$
   (3) $y = -4x + 3$
   (4) $y = 3x - 4$
   (5) I do not know.

7. The equation of the line through the point (-2 3) wiht slope 4 is:
   (1) $y + 3 = 4(x - 2)$
   (2) $y - 3 = 4(x + 2)$
   (3) $4(y - 3) = (x + 2)$
   (4) $4(y + 3) \quad (x - 2)$
   (5) I do not know.

8. The equation of the line with x-intercept 3 and y-intercept 2 is:
   (1) $2x + 3y = 1$
   (2) $3x + 2y = 1$
   (3) $x/2 \cdot y/3 = 1$
   (4) $x/3 + y/2 \quad 1$
   (5) I do not know

9. The eqaation of the line through the points (2,3) and (4 7) is:
   (1) $y - 3 = 4/2(x - 2)$
   (2) $y - 3 = 10/6(x - 2)$
   (3) $y - 3 = -4/2(x - 2)$
   (4) none of these
   (5) I do not know.

10. $y = 2/3\ x + 17$. A line perpendicular to this line would have a slope of:
   (1) 2/3 (2) 3/2 (3) -2/3 (4) -3/2 (5) I do not know

11. Which of the following equations represents a line which is perpendicular to the y-axis?
   (1) $y = x$
   (2) $xy = 1$
   (3) $x = 4$
   (4) $y = 4$
   (5) I do not know

12. The equation of a line through the point (0,5) and perpendicular to the line $y = 2x + 7$ is:
   (1) $y = 2x + 5$
   (2) $y = 1/x + 5$
   (3) $y = -1/2x + 5$
   (4) $y = -2x + 5$
   (5) I do not know.

13. The equation of a line through the point (0 5) and parallel to the line $y = 2x + 7$ is:
   (1) $y = 2x + 5$
   (2) $y = 1/2x + 5$
   (3) $y = -1/2x + 5$
   (4) $y = -2x + 5$
   (5) I do not know.

14. The distance from the point (0,0) to the line $y = 2x + 3$ is:
   (1) 0
   (2) 2
   (3) $3/\sqrt{5}$
   (4) $\sqrt{8}$
   (5) I do not know

15. Given: point P(2,3) and pointQ(7,5). The lenght of the line segment PQ is
   (1) 2
   (2) $\sqrt{29}$
   (3) $\sqrt{21}$
   (4) 5
   (5) I do not know

16. Given $Ax^2 + By^2 + Cx + Dy + E = 0$ and A B not equal to 0.
It is impossible for the above to be:
(1) a circle
(2) an ellipse
(3) a line
(4) a hyperbola
(5) I do not know.

17. $y^2 + Dx + Ey + F = 0$ and D, E and $F \neq 0$. This is a
(1) circle (2) parabola (3) ellipse (4) hyperbola (5) I do not know

18. $\frac{(x-2)^2}{49} + \frac{(y-3)^2}{25} = 1$ is an equation of a(n):
(1) hyperbola
(2) ellipse
(3) parabola
(4) circle
(5) I do not know

19. $9x^2 + 16y^2 - 18x - 64y - 71 = 0$ is an equation of a(n):
(1) ellipse
(2) circle
(3) hyperbola
(4) parabola
(5) I do not know

20. $9x^2 - 16y^2 + 36x + 32y = 124$ is an equation of a(n):
(1) ellipse
(2) hyperbola
(3) parabola
(4) circle
(5) I Do not know.

21. $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is an equation of a(n):

(1) ellipse
(2) circle
(3) hyperbola
(4) parabola
(5) I do not know

22. Conics have equations of a degree no higher than:
(1) 1 (2) 2 (3) 3 (4) 4 (5) I do not know

23. Given $x^2 + (4/3)x +$ _____. Complete the square by filling in the blank. The third term is:
(1) 2/3 (2) 4/9 (3) 16/9 (4) 4/3 (5) I do not know

24. The coordinates of the center of the circle represented by $x^2 - 4x + y^2 = 5$ are:

(1) (4,5)
(2) (4 0)
(3) (2 0)
(4) (-2,0)
(5) I do not know

25. The equation of the circle with center at the point (2,4) and passing through the origin is:

(1) $(x - 2)^2 + (y - 4)^2 = 20$
(2) $(x - 2)^2 + (y - 4)^2 = \sqrt{20}$
(3) $(x - 2)^2 + (y - 4)^2 = 4$
(4) none of these
(5) I do not know

26. Given a circle whose center is (6,-2) and a radius of 4. Its equation is:

(1) $(x + 6)^2 + (y - 2)^2 = 4$
(2) $(x + 6)^2 + (y - 2)^2 = 16$
(3) $(x - 6)^2 + (y + 2)^2 = 4$
(4) $(x - 6)^2 + (y + 2)^2 = 16$
(5) I do not know

27. The equation of the circle in #26 is:

(1) $x^2 + y^2 + 12x - 4y + 24 = 0$

(2) $x^2 + y^2 - 12x + 4y + 24 = 0$

(3) $x^2 + y^2 + 12x - 4y - 24 = 0$

(4) $x^2 + y^2 - 12x + 4y - 24 = 0$

(5) I do not know

28. Given circle A: $x^2 + y^2 = 4$ and circle B: $(x - 2)^2 + (y - 4)^2 = 4$

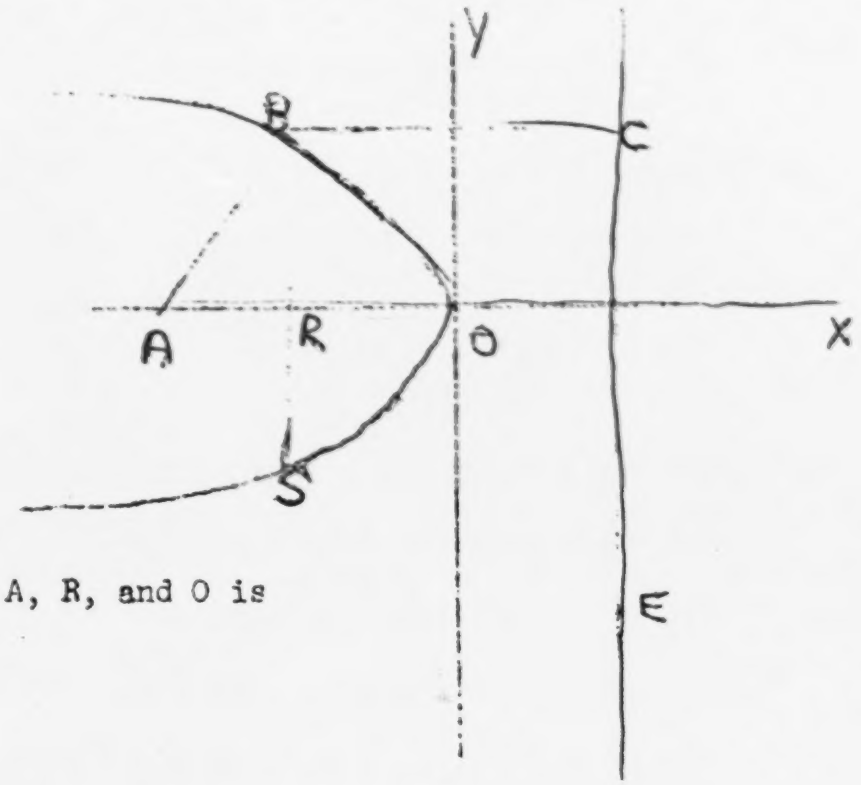
(1) A is greater in area than B.
(2) B is greater in area than A.
(3) A is the same area as B.
(4) The center of B is the point (-2, -4)
(5) I do not know

29. Given a line L, and a point P. The locus of all points equidistant from L and P is:

(1) a circle
(2) an ellipse
(3) a parabola
(4) hyperbola
(5) I do not know.

30. $x^2 = 6y$. This is
(1) a parabola opening upward
(2) a parabola opening downward
(3) a parabola opening to the right
(4) a hyperbola
(5) I do not know

In the sketch at the right, any pt. B on the curve is equidistant from A and from the line CE. The dotted line is parallel to the y-axis. BS is perpendicular to the x-axis and BR RS. Questions 31,32, 33 refer to the sketch at the right.



31. The figure is a(n)
(1) ellipse
(2) parabola
(3) circle
(4) hyperbola
(5) I do not know

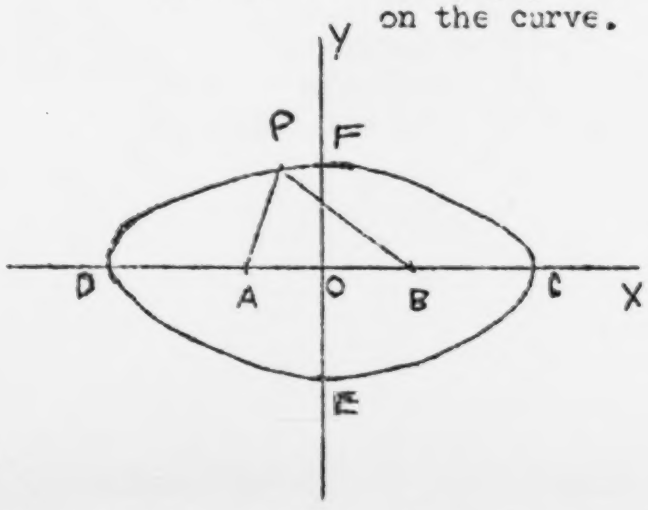
32. The line CE is
(1) an axis of symmetry
(2) a directrix
(3) the focal chord
(4) the latus rectum
(5) I do not know

33. The line containing the points A, R, and O is
(1) an axis of symmetry
(2) a directrix
(3) the focal chord
(4) an asymptote
(5) I do not know

34. Given: a plane and a locus of a point that remains in the plane. The point so that the sum of the distances from two fixed points is constant. This locus is a(n):
(1) parabola
(2) circle
(3) ellipse
(4) hyperbola
(5) I do not know

Questions 35, 36, and 37 refer to the sketch at the right.

AP + PB = 4 for any point on the curve.



35. The point A is a(n):
(1) vertex
(2) center
(3) focus
(4) median
(5) I do not know

36. The point ● is a(n):
(1) vertex
(2) center
(3) focus
(4) median
(5) I do not know

37. The line segment DC is
(1) asymptote
(2) minor axis
(3) major axis
(4) directrix
(5) I do not know

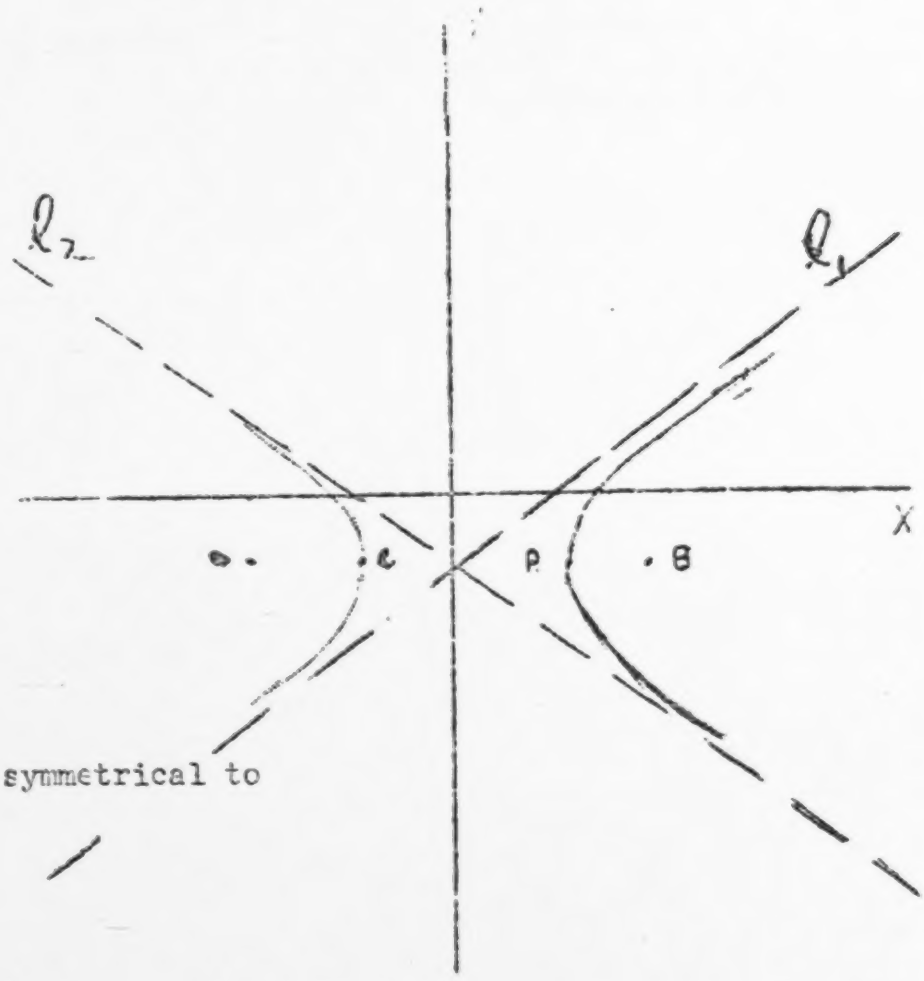
38. Given: a plane and a locus of a point that remains in the plane. The point moves so that the difference of the distances from two fixed points is constant. The locus is a(n):
(1) parabola
(2) circle
(3) ellipse
(4) hyperbola
(5) I do not know

Questions 39 40, 41 refer to the sketch at the right.

39. Line $\overleftrightarrow{k}$ is a(n):
(1) directrix
(2) transverse axis
(3) asymptote
(4) conjugate axis
(5) I do not know

40. Point B is
(1) center
(2) vertex
(3) an asymptote
(4) a focus
(5) I do not know

41. Point A is a(n):
(1) focus
(2) vertex
(3) center
(4) asymptote
(5) I do not know

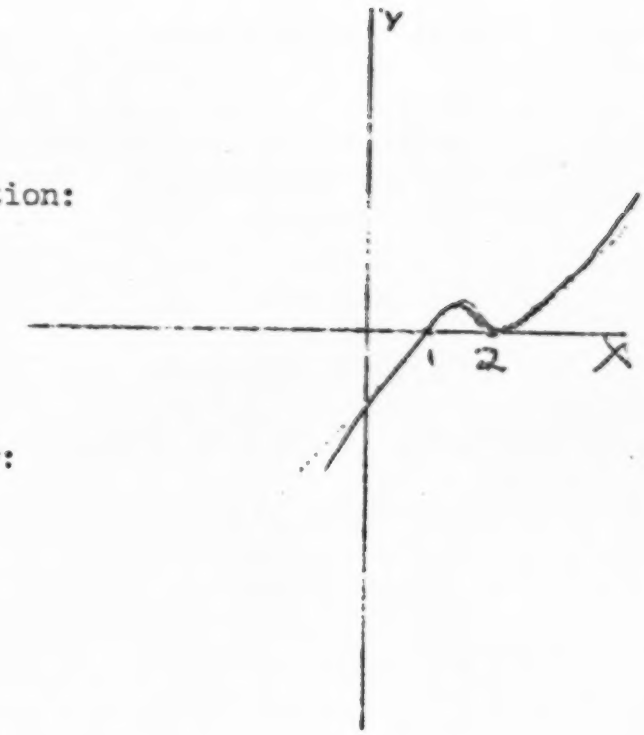


42. xy = 40 is a conic. It is symmetrical to
(1) the x-axis
(2) the y-axis
(3) the origin
(4) I do not know

43. If $y = \frac{x + 2}{x - 2}$ then at $x = 2$ the graph:
(1) would be continuous
(2) would have a horizontal asymptote
(3) would have a vertical asymptote
(4) would have a slope of 2
(5) I do not know

44. A sketch of the equation $y = \frac{2x}{x^2 - 4}$ would have:
(1) no asymptote
(2) one asymptote
(3) two horizontal asymptotes
(4) two vertical asymptotes
(5) I do not know

45. The sketch at the right represents the equation:
(1) $(x - 1)(x - 2) = 0$
(2) $(x + 1)(x + 2) = 0$
(3) $(x - 1)(x - 2)^2 = 0$
(4) $(x + 1)(x + 2)^2 = 0$
(5) I do not know



46. $x^3 + 1 = 3xy$. The sketch of this would show:
(1) symmetry with respect to the x-axis
(2) symmetry with respect to the y-axis
(3) symmetry with respect to the origin
(4) none of the above symmetries
(5) I do not know

**ANALYTIC GEOMETRY PRETEST AND RESOURCE MATERIALS*

ANS. KEY 4231212414431323221232231423312213323434233434
1124404** 4133411243431254423343555253142312123234232343

16 RIGHT 25 WRONG 5 DONT KNOW

FORMS OF STRAIGHT LINE EQUATION
2 RIGHT 3 WRONG 0 DONT KNOW OUT OF 5
I. DAVIS P181-203 REES P328-331 FAWCETT P42-51

WRITING EQUATIONS OF STRAIGHT LINE
1 RIGHT 3 WRONG 0 DONT KNOW OUT OF 4
T. DAVIS P92-107 REES P328-331 FAWCETT P308-311

PARALLEL AND PERPENDICULAR LINES
3 RIGHT 1 WRONG 0 DONT KNOW OUT OF 4

DISTANCE FORMULA
0 RIGHT 1 WRONG 1 DONT KNOW OUT OF 2
T. DAVIS P49-91

IDENTIFY CONICS
1 RIGHT 6 WRONG 0 DONT KNOW OUT OF 7
I.DAVIS P133-147,206-209,397-412

CIRCLE
1 RIGHT 1 WRONG 4 DONT KNOW OUT OF 6
T. DAVIS P209-233 FAWCETT P333-345 REES P332-336

PARABOLA
2 RIGHT 3 WRONG 0 DONT KNOW OUT OF 5
T. DAVIS P234-265 FAWCETT P346-356 REES P345-348

ELLIPSE
2 RIGHT 2 WRONG 0 DONT KNOW OUT OF 4
T. DAVIS P266-314 REES P349-352 FAWCETT P359-370

HYPERBOLA
3 RIGHT 1 WRONG 0 DONT KNOW OUT OF 4

SYMMETRIES AND ASYMPTOTES
1 RIGHT 4 WRONG 0 DONT KNOW OUT OF 5
T. DAVIS P148-180 REES P338-342

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                        ANALYTIC GEOMETRY INPUT FOR INFOS
// JOB    CCC1
// XEQ INFCS
  46  10  4
   5   4   4   2   7   6   5   4   4   5
   3   3   3   2   5   4   3   3   3   3
     1     2     3     4     5
FORMS OF STRAIGHT LINE EQUATION
T. DAVIS P181-203      REES P328-331      FAWCETT P42-51

     6     7     8     9
WRITING EQUATIONS OF STRAIGHT LINE
T. DAVIS P92-107       REES P328-331      FAWCETT P308-311

    10    11    12    13
PARALLEL AND PERPENDICULAR LINES
T. DAVIS P108-116      REES P324-327

    14    15
DISTANCE FORMULA
T. DAVIS P49-51

    16    17    18    19    20    21    22
IDENTIFY CONICS
T.DAVIS P133-147,206-209,397-412

    23    24    25    26    27    28
CIRCLE
T. DAVIS P209-233      FAWCETT P333-345      REES P332-336

    29    30    31    32    33
PARABOLA
T. DAVIS P234-265      FAWCETT P346-356      REES P345-348

    34    35    36    37
ELLIPSE
T. DAVIS P266-314      REES P349-352      FAWCETT P359-370

    38    39    40    41
HYPERBOLA
T. DAVIS P315-358      REES P353-355      FAWCETT P373-383

    42    43    44    45    46
SYMMETRIES AND ASYMPTOTES
T. DAVIS P148-180      REES P338-342

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